

EXAM 1 - MATH 112

DATE: Friday, January 27

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 3 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Find the following limits:

(a) $\lim_{x \rightarrow 7} \frac{2x^2 - 13x - 7}{x^2 - 2x - 35}$ (1 point)

(b) $\lim_{x \rightarrow 2} \frac{x-2}{\sqrt{x+2}-2}$ (2 points)

2. (a) Give the *formal* (not geometric) definition of a function $y = f(x)$ being continuous at $x = a$. (1 point)

(b) Consider the function

$$f(x) = \begin{cases} 5x - \frac{9}{2}, & \text{if } x < 1 \\ 2, & \text{if } x = 1 \\ \frac{\sqrt{x}-1}{x-1}, & \text{if } x > 1 \end{cases}$$

Does $\lim_{x \rightarrow 1} f(x)$ exist? Explain in detail. (1 point) Is $f(x)$ continuous at $x = 1$? Explain in detail. (1 point)

3. (a) The limit definition of the derivative says that $f'(a) = ?$ (1 point)

(b) Use the limit definition of the derivative to compute the derivative of the function $f(x) = 3x - \frac{2}{x}$ at the point $x = 2$. (2 points)

4. The height h in feet of an object fired straight up from the ground with an initial velocity of 640 feet per second is given by $h(t) = -16t^2 + 640t$, where t is time in seconds.

(a) Find the maximum height that the object will reach and when it will reach that maximum height. (2 points)

(b) Find when the object will hit the ground. (1 point)

5. Find the derivatives of the functions:

(a) $f(x) = (x^4 + \frac{3}{x})(\sqrt[4]{x^7} - \frac{2}{\sqrt[3]{x^2}})$ (2 points)

(b) $g(x) = \frac{x^5 + 7x - 2}{7 - x^9}$ (1 point)

6. (a) Find the derivative of the function $f(x) = (x + 2)(\frac{x-5}{x+1})$. (1.5 points)

(b) Find the slope of the tangent line to the graph of $y = f(x)$ at $x = 0$. (0.5 points)

(c) Find the equation to the tangent of $y = f(x)$ at $x = 0$. (1 point)