

TEST 11 - MATH 140

DATE: Friday, March 31

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) Sketch the graph of the function  $y = \cot x$  in the interval  $0 < x < \pi$ . (1 point)  
(b) Sketch the graph of the inverse  $y = \cot^{-1} x$  based on the previous part. (2 points)  
(c) Find the exact value of the expression  $\cot^{-1}(\cot \frac{13\pi}{2})$ . (2 points)
2. (a) Establish the identity  $1 - \frac{\sin^2 \theta}{1 - \cos \theta} = -\cos \theta$ . (2.5 points)  
(b) Establish the identity  $\sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta$ . (2.5 points)
3. (a) Suppose  $\alpha = \sin^{-1} \frac{4}{7}$ . Find  $\sin \alpha$  and  $\cos \alpha$ . (1 point).  
(b) Suppose  $\beta = \tan^{-1}(-\frac{5}{12})$ . Find  $\sin \beta$  and  $\cos \beta$ . (2 points)  
(c) Find  $\sin(\alpha - \beta)$ . (2 points)
4. Compute  $\sec(\tan^{-1} \frac{3}{4} + \sin^{-1}(-\frac{12}{13}))$ . I want to see **all** the details **neatly written** down. (5 points).
5. Establish the identity

$$\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} - \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} = 2 \tan(2\theta). \quad (5 \text{ points})$$