TEST 13 - MATH 140

DATE: Friday, April 13

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) Consider the point with polar coordinates $(-4, \frac{5\pi}{6})$. Find a polar representation (r, θ) of the point, such that
   i. $r > 0, 0 \leq \theta < 2\pi$ (1 point)
   ii. $r < 0, -2\pi \leq \theta < 0$ (1 point)
   (b) Convert the point $(5, -5\sqrt{3})$ into polar coordinates. (2 points)
   (c) Convert the point $(-4, \frac{5\pi}{6})$ into cartesian coordinates. (1 point)

2. Consider the polar equation $r = -3\sin\theta$.
   (a) Convert the given equation to cartesian coordinates. (2 points)
   (b) Manipulate the equation that you obtained in the previous part so that you may be able to identify its graph. (2 points)
   (c) Sketch its graph on the plane. (1 point)

3. (a) Suppose that $z = 5 - 12i$. Graph z on the complex plane. (1 point)
   (b) Find $|z|$. (1 point)
   (c) Find $\overline{z}$. (1 point)
   (d) Show that, if $z = a + bi$, then $|z|^2 = z\overline{z}$. (You should do this for $z = a + bi$; Not the specific $z = 5 - 12i$ of the previous parts.) (2 points)

4. Suppose that $z = -3 + 3\sqrt{3}i$ and $w = -4i$.
   (a) Write z in polar form. (1 point)
   (b) Write w in polar form. (1 point)
   (c) Multiply zw and leave your answer in polar form. (1 point)
   (d) Divide $\frac{z}{w}$ and leave your answer in polar form. (1 point)
   (e) Find z^6 and write it in standard form. (1 point)

5. Find the six complex 6-th roots of $z = -i$. Regardless of the method that you use to identify then, please, present your work and your answers **clearly!** (5 points)