EXAM 1 - MATH 216

Thursday, February 3, 2006

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Read each problem very carefully before starting to solve it. Each question is worth 2 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Give a formula and a proof for the number of r-combinations from a set with n elements when repetition of elements is allowed. (2 points)
2. **Prove via a combinatorial argument** that $\binom{11}{4} = \sum_{k=3}^{10} \binom{k}{3}$. (2 points)
3. Find the coefficient of
   (a) x^9 in $(2-x)^{19}$. (1 point)
   (b) How many terms are there in the expansion of $(x+y+z+w)^{27}$? (0.5 points)
   (c) What is the coefficient in the above expansion of the term $x^3y^8z^{10}w^6$? (0.5 points)
4. (a) Five men, five women and five children are assigned to stand around a circular fountain. Find the number of such assignments if a man is to be sided by a woman and a child, a woman by a man and a child and a child by a man and a woman. (1 point)
   (b) When a die is rolled, one of the first six positive integers is obtained. Suppose that the die is rolled nine times and the sum of the nine integers thus obtained is added. The nine throws constitute a trial. Find the number of possible trials such that the sum is at most 15. (1 point)
5. (a) How many ways are there to distribute 12 indistinguishable balls into six distinguishable bins? (1 point)
   (b) How many ways are there to distribute 15 distinguishable objects into five distinguishable boxes so that the boxes have one, two, three, four and five objects in them, respectively? (1 point)