EXAM 2 - MATH 216

Friday, February 24, 2006

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 2 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Show that any sequence of $n^2 + 1$ distinct numbers contains a subsequence of at least $n + 1$ terms which is either an increasing or a decreasing sequence. (2 points)

2. Give and prove the formula for the number of derangements of a set of cardinality n. (2 points)

3. (a) Find the number of elements in the set $\{1, 2, \ldots, 200\}$ that are not divisible by 2 or 3 or 5. (1 point)

   (b) Find the number of solutions in integers of the linear equation $x + y + z = 20$ if $2 \leq x \leq 5$, $4 \leq y \leq 8$ and $1 \leq z \leq 10$. (1 point)

4. (a) Show that in a strictly increasing sequence of 50 positive integers, none of which exceeds 90, there exists at least one pair whose difference is equal to 9. (1 point)

   (b) In how many ways can a party of 8 people sit in 8 designated seats of a row in a movie theater if a couple that has just split up insist on not sitting side-by-side? (1 point)

5. (a) A group of 15 executives are to share 5 secretaries. Each executive is assigned exactly 1 secretary and no secretary is assigned to more than 4 executives. Prove that at least 3 secretaries are assigned to 3 or more executives. (1 point)

   (b) A combination lock requires three selections of numbers, each from 1 through 39. Suppose the lock is constructed in such a way that no number can be used twice in a row but the same number may occur both first and third. How many different combinations are possible? (1 point)