

EXAM 3 - MATH 216

Friday, March 31, 2006

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Read each problem very carefully before starting to solve it. Each question is worth 2 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

- Use generating functions to provide formulas for
 - the r -combinations of a set of n elements. (0.5 points)
 - the r -collections of a set of n elements. (0.75 points)
 - the r -sequences (r -permutations with repetitions) of a set of n -elements. (0.75 points)
- Show that, if $n, k > 0$ are integers, then $\binom{-n}{k} = (-1)^k \binom{n+k-1}{k}$. (1 point)
 - Show that $(1 - 2x)^{-3/2}$ is the exponential generating function of the sequence $1, 1 \cdot 3, 1 \cdot 3 \cdot 5, 1 \cdot 3 \cdot 5 \cdot 7, \dots$ (1 point)
- Use generating functions to find the number of ways that Professor Boger can allocate
 - thirteen new identical DELL[®] computers to three distinct computer labs so that each lab gets at least two new computers. (1 point)
 - ten distinct courses to four faculty members so that each faculty member gets at least one course. (1 point)
- Use generating functions to find the number of ways to choose a dozen bagels from three varieties-egg, salty and plain-if at least two bagels of each kind but no more than three salty bagels are chosen. (2 points)
- In the older days of the Campionato[®] (the Italian soccer championship) a team earned zero points for a loss, one point for a tie and two points for a win (nowadays it is three points for a win). Use generating functions to find the number of ways in which a team could end up having 7 points in the Campionato table after having played its first twelve games according to the old rules. (1 point)
 - Use generating functions to prove Pascal's identity $\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$. (1 point)