EXAM 4 - MATH 216
Friday, April 21, 2006
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Read each problem very carefully before starting to solve it. Each question is worth 2 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Write down a recurrence relation for the sum s_n of the first n natural numbers. Then solve the recurrence by using both the characteristic equation method (1 point) and the generating function method (1 point).

2. Consider the nonhomogeneous recurrence relation $a_n = 5a_{n-1} - 4a_{n-2} + f(n)$. Find a general solution of this recurrence in the following three cases:

   (a) $f(n) = 3n + 2$. (0.75 points)

   (b) $f(n) = 3 \cdot 4^n$. (0.75 points)

   (c) $f(n) = 7 \cdot 5^n$. (0.5 points)

3. Solve the linear homogeneous recurrence with constant coefficients of order two $a_n = 4a_{n-1} - 8a_{n-2}$, with initial conditions $a_0 = 1$ and $a_1 = 2$. (2 points)

4. Find a recurrence relation for the number a_n of the n-digit decimal strings that contain an even number of 0's. (0.5 points) Then find the generating function for the sequence $\{a_n\}_{n=0}^{\infty}$. (0.75 points) Finally, use the generating function to find a_n. (0.75 points)

5. Find a recurrence relation for the number a_n of ways to lay out a walkway with slate tiles if the tiles are red, green or gray, so that no two red tiles are adjacent and tiles of the same color are considered indistinguishable. Then solve the recurrence to find an expression for a_n. (2 points)