

## HOMEWORK 2 - MATH 216

DUE DATE: When Chapter 1 has been covered.

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

- 1.73 Show that it is possible to have a set of 5 people such that there is no subgroup of 3 strangers or a subgroup of 3 people known to one another in this set.
- 1.77 Show that in every finite set of numbers there is a number that is greater than or equal to the arithmetic mean of the numbers in the set.
1. Let  $d$  be a positive integer. Show that among any group of  $d + 1$  integers there are two with exactly the same remainder when divided by  $d$ .
2. What is the minimum number of students, each of whom comes from one of the 50 states, enrolled in a university to guarantee that there are at least 100 who come from the same state?
3. Let  $(x_i, y_i)$ ,  $i = 1, 2, 3, 4, 5$ , be a set of five distinct points with integer coordinates in the  $xy$ -plane. Show that the midpoint of the line joining at least one pair of these points has integer coordinates.
4. How many numbers must be selected from the set  $\{1, 3, 5, 7, 9, 11, 13, 15\}$  to guarantee that at least one pair of these numbers add up to 16?
5. Find an increasing subsequence of maximal length and a decreasing subsequence of maximal length in the sequence 22, 5, 7, 2, 23, 10, 15, 21, 3, 17.
6. Find the least number of cables required to connect eight computers to four printers to guarantee that four computers can directly access four different printers. Justify your answer.
7. Prove that at a party where there are at least two people, there are two people who know the same number of other people there.
8. Find the number of positive integers not exceeding 100 that are not divisible by 5 or by 7.
- 1.78 Let  $X = \{1, 2, 3, \dots, 600\}$ . Find the number of elements in  $X$  that are not divisible by 3 or 5 or 7.
9. How many bit strings of length 8 do not contain six consecutive 0's?
10. How many terms are there in the formula for the number of elements in the union of 10 sets given by the principle of inclusion and exclusion?

11. What is the probability that none of 10 people receives the correct hat if a hatcher person hands their hats back randomly?
12. A group of  $n$  students is assigned seats for each of two classes in the same classroom. How many ways can these seats be assigned if no student is assigned the same seat for both classes?
- 1.83 Use a combinatorial argument to establish the identity:

$$C(n, 0)D_n + C(n, 1)D_{n-1} + C(n, 2)D_{n-2} + \cdots + C(n, n)D_0 = n!.$$

13. How many solutions does the equation  $x_1 + x_2 + x_3 = 13$  have, where  $x_1, x_2, x_3$  are nonnegative integers less than 6?
- 1.84 Find the number of solutions in integers of the linear equation  $p + q + r = 25$ , where  $2 \leq p \leq 4$ ,  $3 \leq q \leq 6$  and  $4 \leq r \leq 8$ .
- 1.86 There are 5 job openings in an office. On the basis of a written test and a personal interview, 4 candidates were selected and each candidate is offered one of the available jobs. Find the number of ways of assigning these jobs to the candidates.