Theory to learn for first exam.

George

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Two of the following questions will be on the exam:

1. State and prove the formula giving the number of r-permutations of an n-element set and the formula for the number of circular permutations of an n-element set.

   Hint: Bottom of page 38 and bottom of page 39 but **for general** n.

2. Give the formula and the proof for the number of r-combinations of a set with n elements.

   Hint: Towards the bottom of page 42.

3. Give both an algebraic and a combinatorial proof of Pascal's identity.

   Hint: Algebraic done in class and combinatorial Theorem 1.3.2 in your book.

4. State and prove both the permutation and the combination allocation problems.

   Hint: Top of page 39 for permutations and middle of page 43 for combinations.

5. Prove that the number of different permutations of n objects, where there are n_1 indistinguishable objects of type 1, n_2 indistinguishable objects of type 2, $\ldots$, and n_k indistinguishable objects of type k, is $\frac{n!}{n_1!n_2!\ldots n_k!}$. Show this in two ways: first using permutations and then using combinations.

   Hint: The proof via permutations is at the bottom of page 40. That using combinations was done in class.

6. State and prove via a combinatorial argument Vandermonde's Identity.

   Hint: Done in class.

7. Use a combinatorial argument to show that $\binom{n+1}{r+1} = \sum_{j=r}^{n} \binom{j}{r}$.

   Hint: Done in class.

8. Give a formula and a proof for the number of r-permutations and for the number of r-combinations of a set with n elements when repetition of elements is allowed.

   Hint: Bottom of page 48 and Theorem 1.4.1, respectively.

In addition, **two problems** out of your **first homework set up to Problem 1.68** plus one **wild-card problem** will be chosen to complete the five questions on your exam.