Theory to learn for second exam.

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Two of the following questions will be on the exam:

1. Show that, if $m = p_1 + p_2 + \ldots + p_n - n + 1$ pigeons are allotted to n pigeonholes, then the first pigeonhole has at least p_1 pigeons or the second pigeonhole has at least p_2 pigeons or $\cdots$ or the n-th pigeonhole has at least p_n pigeons.

   Hint: Bottom of page 55. However, the book's argument is not complete because it uses Part (a). The argument we saw in class is more transparent.

2. Suppose that a computer science lab has 15 workstations and ten servers. A cable can be used to directly connect a work station to a server. Only one direct connection to a server can be active at any one time. We want to guarantee that at any time any set of ten or fewer workstations can simultaneously access different servers via direct connections. What is the minimum number of direct connections needed to achieve this goal?

   Hint: Done in detail in class.

3. During a month with 30 days a baseball team plays at least one game a day, but no more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games.

   Hint: Done in detail in class.

4. Let $X = \{1, 2, \ldots, 2n\}$ and S be any subset of X with $(n + 1)$ elements. Then there are at least two numbers in S such that one divides the other.

   Hint: Page 57.

5. Any sequence of $n^2 + 1$ distinct numbers contains a subsequence of at least $n + 1$ terms which is either an increasing sequence or a decreasing sequence.

   Hint: Page 58.

6. State and prove the general form of the Inclusion-Exclusion Principle.

   Hint: Theorem 1.6.1 on page 61.

7. Show that $\phi(60) = \frac{60(2-1)(3-1)(5-1)}{2 \cdot 3 \cdot 5}$ using the Inclusion-exclusion Principle.

   Hint: Example 1.6.4 on page 63.

8. Give and prove the formula for the total number D_n of derangements of a set of cardinality n.

   Hint: Theorem 1.6.3 on page 65.

9. Show that the number of r-sequences that can be formed using the elements of a set with n elements such that in every such sequence each element of the set appears at least once is $n!S(r, n)$, where $S(r, n)$ is the Stirling number of the second kind.

   Hint: Theorem 1.6.4 on page 66. However the proof is not complete. The complete proof was presented in class.

In addition, **two problems** out of your **second homework set** plus one **wild-card problem** will be chosen to complete the five questions on your exam.