Theory/Problems to learn for fourth exam.

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Two of the following questions will be on the exam:

1. Solve the Fibonacci recurrence $a_n = a_{n-1} + a_{n-2}$ with initial conditions $a_0 = a_1 = 1$.

   Hint: Done in detail in class.

2. Let n ovals be drawn on the plane such that they intersect pairwise at exactly two points and no three of them meet at the same point. Formulate and solve a recurrence relation for the number of regions into which these n ovals divide the plane.

   Hint: Done in detail in class.

3. Show that the basic solution corresponding to two complex conjugate roots $z = a + bi$ and $\overline{z} = a - bi$ of the characteristic equation of a recurrence relation may be written in the form $b_n = (A+B)\sqrt{a^2+b^2}^n \cos(n \tan^{-1} \frac{b}{a}) + i(A-B)\sqrt{a^2+b^2}^n \sin(n \tan^{-1} \frac{b}{a})$.

   Hint: Done in detail in class.

4. Describe formally in detail the process by which one identifies the particular solutions of the nonhomogeneous recurrence relation $a_n = c_1 a_{n-1} + \cdots + c_r a_{n-r} + f(n)$ in the two special cases when $f(n) = c \cdot q^n$ and $f(n) = c \cdot n^k$.

   Hint: Bottom of page 102 in your book. Also presented in detail in class.

5. Formulate a recurrence relation that solves the problem of finding the sum s_n of the first n nonnegative integers. Then solve the recurrence using the method of generating functions.

   Hint: We solved the recurrence in class using the characteristic equation. It is not difficult to apply the generating function method.

6. Formulate from scratch a recurrence relation solving the problem of finding the number a_n of the n-digit quaternary sequences that contain an even number of 0's. Then solve the recurrence using the method of generating functions.

   Hint: Done in detail in class.

7. Find the generating function $A(x)$ that solves the Fibonacci recurrence $a_n = a_{n-1} + a_{n-2}$, with $a_0 = a_1 = 1$. Then use $A(x)$ to find a_4.

   Hint: It is not difficult to do this using the generating function method. Note that **you do not have to find** a_n. **You only need** a_4.

In addition, **two problems** out of your **third homework set** plus one **wild-card problem** will be chosen to complete the five questions on your exam.