HOMEWORK 4 - MATH 112

DUE DATE: Monday, February 19

INSTRUCTOR: George Voutsadakis

Read each problem **very carefully** before starting to solve it. Four out of the eight problems will be chosen at random and graded. Each problem graded is worth 3 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

- 1. A spherical balloon is inflated with gas at a rate of 20 cubic feet per minute. How fast is the radius of the balloon changing at the instant when the radius is 2 feet?
- 2. A 25-foot ladder is leaning against a house. The base of the ladder is pulled away from the house at the rate of 2 feet per second. How fast is the top of the ladder sliding down the wall when the base is 7 feet from the house?
- 3. Find the domain, the critical numbers and the open intervals on which the given function is increasing or decreasing.

(a)
$$f(x) = x^3 - 6x^2$$

(b)
$$g(x) = \sqrt{4 - x^2}$$

(c)
$$f(x) = \frac{2x}{16-x^2}$$

4. Create the table for the first derivative in order to find the monotonicity and the relative extrema of the following functions.

(a)
$$f(x) = -4x^2 + 4x + 1$$

(b)
$$g(x) = \frac{1}{5}x^5 - x$$

(c)
$$h(x) = x^4 - 32x + 4$$

5. Find the absolute extrema of the function in the given closed interval:

(a)
$$f(x) = x^3 - 12x$$
 on $[0, 4]$

(b)
$$g(x) = \frac{x}{x-2}$$
 on [3, 5]

6. Use the second derivative test to find all relative extrema of the function. Write "Not Applicable" when the test is inconclusive.

(a)
$$f(x) = x^4 - 4x^3 + 2$$

(b)
$$g(x) = x + \frac{4}{x}$$

(c)
$$h(x) = \frac{x}{x^2 - 1}$$

7. Make a sign table for both the first and the second derivative to find both the intervals of monotonicity and the intervals of concavity of the following functions.

(a)
$$f(x) = (x-1)^3(x-5)$$

(b)
$$g(x) = (1-x)(x-4)(x^2-4)$$

8. A rectangular page is to contain 30 square inches of print. The margins at the top and at the bottom are to be 2 inches wide. The margins on each side are to be 1 inch wide. Find the dimensions of the page such that the least amount of paper is used.

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