

YOUR NAME: \_\_\_\_\_

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. The number  $N$  of deer present at time  $t$  (measured in years since the herd was introduced) on a certain wildlife reserve has been determined by ecologists to be given by the function

$$N(t) = \frac{12.36}{0.03 + 0.55t}.$$

- (a) How many deer were initially on the reserve?
- (b) Calculate  $N(10)$  and explain in a short and precise sentence the meaning of the number that you calculated.
- (c) How much increase in the deer population do you expect from the 10th to the 15th year?
- (d) What is the average rate of increase of the deer population between the 3rd and the 8th year?

2. Suppose a piece of aluminum is removed from an oven and is left to cool. The following table gives the temperature  $A = A(t)$  in degrees Fahrenheit of the aluminum  $t$  minutes after it is removed from the oven.

$t = \text{Minutes}$	0	30	60	90	120	150	180	210
$A = \text{Temperature}$	302	152	100	81	75	73	72	72

- (a) Explain the meaning of  $A(75)$  and estimate its value.
- (b) What is the average rate of change per minute of the temperature during the first half-hour of cooling?
- (c) What is the average rate of change per minute of the temperature during the first half of the second hour of cooling?
- (d) Use functional notation to express the temperature of the aluminum after 1 hour and 13 minutes. Estimate the temperature at that time.
- (e) What is the temperature of the oven and what is the room temperature? Please, explain your answer with short precise phrases.

3. Consider the function

$$f(x) = 3(x - 2)^2 e^{-x}$$

in the interval  $[1, 7]$ . Our goal is to study in detail the graph  $y = f(x)$ . Please, follow the directions given closely and answer the following questions precisely.

(a) Graph  $y = f(x)$  with your calculators setting your window for  $x$  between 1 and 7 and for  $y$  between  $-0.2$  and  $1.2$ . Reproduce your graph **carefully** here. If the shape is not very clear, you might want to zoom in and out to explore some details before drawing.

(b) What is the initial value for  $f$  in  $[1, 7]$ ? If the graph is allowed to evolve further to the right outside  $[1, 7]$  does it have a limiting value?

(c) Find the intervals of monotonicity (intervals where graph is increasing/decreasing) and the local extrema in  $[1, 7]$ . Summarize your results in a small table.

(d) Find the intervals of concavity (intervals where graph is concave up/concave down) and the inflection points in  $[1, 7]$ . Summarize your results in a small table.

4. Suppose that a manufacturer of widgets has fixed costs of \$9,000 per month and that the variable cost is \$ 15 per widget. Suppose, moreover, that the manufacturer sells the widgets for \$25 each.
- (a) Find a formula expressing the total cost  $C$  of this manufacturer in a month as a function of the number  $N$  of widgets produced in a month. State the units used.
  
  
  
  
  
  
  
  
  
  
  - (b) Express using functional notation the total cost if there are 250 widgets produced in a month and then calculate its value.
  
  
  
  
  
  
  
  
  
  
  - (c) Use a formula to express the manufacturer's total revenue  $R$  in a month as a function of the number  $N$  of widgets produced in a month. State the units used.
  
  
  
  
  
  
  
  
  
  
  - (d) Use a formula to express the profit  $P$  of the manufacturer as a function of the number  $N$  of widgets produced in a month.
  
  
  
  
  
  
  
  
  
  
  - (e) Express using functional notation the profit of the manufacturer if there are 250 widgets produced in a month and then calculate its value.
  
  
  
  
  
  
  
  
  
  
  - (f) How many widgets does the manufacturer need to produce to break even?