EXAM 3 - MATH 111 Friday, November 4

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. The element Uranium 239 is an unstable isotope of uranium that decays rapidly. 1 gram of Uranium 239 is placed in a container and the remaining amount is measured at 1 minute intervals. The following table records the measurements:

Time in Minutes	0	1	2	3	4
Grams Remaining	1	0.971	0.943	0.916	0.889

   (a) Show that remaining mass versus time is exponential. Please, explain.

   (b) What is the decay rate per minute? What is the the percentage decay each minute?

   (c) What is the hourly decay rate? (Explain)

   (d) Use functional notation to express the amount of Uranium 239 remaining after 10 minutes and calculate its value.

   (e) What is the half life of Uranium 239 (i.e., how long does it take for a quantity of the element to be reduced in half)? Show your work.

2. The maximum length that a haddock could be expected to grow is about 53 cm. Let $D = D(t)$ denote the difference between 53 cm and the length of a haddock at t years of age. The following table summarizes experimentally collected values of D:

Age t	2	5	7	13	19
Difference D	28.2	16.1	9.5	3.3	1.0

(a) Find an exponential model of D as a function of t. Please, explain.

(b) Find a model for the length $L = L(t)$ in centimeters of a haddock at age t years old. Please, explain.

(c) Plot the data points for the length L of the haddock at ages 2, 5, 7, 13 and 19 together with the graph of the model for L vs. t. Please, label your axes carefully.

(d) A fisherman caught a haddock that measures 41 cm. What is its approximate age? Explain.

3. An animal grows according to the formula $L = 0.6 \log(2 + 5T)$, where L is the length in feet and T is the age in years.

   (a) Draw the graph of length versus age, including age up to 20 years. Please, label carefully your axes.

   (b) Describe how the length is changing with age.

   (c) Explain in practical terms the meaning of $L(15)$ and, then, calculate its value.

   (d) How old is the animal when it is 1 foot long? Explain.

   (e) Use a formula to express the age T as function of the length L of the animal. Show all work.

4. For a certain organism, the daily rate of gain in weight G as a function of energy intake M in its first month of life is modeled by

$$G = 3 + 2\log M$$

where G and M are measured in appropriate units.

(a) If the daily energy intake is 0.5 units, what is the daily rate of gain in weight?

(b) A zookeeper would like to feed a baby animal of the species so as to maintain daily rate of gain in weight at the level of 0.03 units. How much energy intake must she ensure for the animal?

(c) How is the daily rate of gain in weight affected when the energy intake of the animal is multiplied by 10?

(d) A baby of the species is gaining weight daily 2 times faster than another animal. How do their daily energy intakes compare?