

YOUR NAME: \_\_\_\_\_

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Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

This problem has a two-fold purpose. The first part will help you familiarize yourself with the **factorial function** and the second will ask you to use it to perform specific tasks by inputting a function and looking at your table of values and/or your graph.

1. (a) Given a nonnegative integer  $n$ , the quantity  $n$  **factorial**, denoted  $n!$ , is defined as follows:

$$n! = n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot 1.$$

For instance  $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ . Compute by hand the values  $4!$  and  $5!$ .

- (b) Suppose that you wanted to use your calculator to compute  $12!$  (which is too big to compute by hand). You write 12, then go to  $\langle \text{MATH} \rangle$ , choose  $\langle \text{PRB} \rangle$  from the list of horizontal choices and and, finally, you pick  $\langle 4 : ! \rangle$  from the list of your vertical choices. Then press  $\langle \text{ENTER} \rangle$  twice and compute. Write your answer here:

- (c) Now you should know how to input the factorial in your calculator. Input  $Y_1 = X!$  and adjust your table with initial value 0 and step 1. Write the first 10 values in your table here:

$X$	
$Y_1$	

2. The number  $C$  of ways to select a group of  $k$  things from a group of  $n$  things is given by

$$C = \frac{n!}{k! \cdot (n - k)!} \quad (\text{assuming that } k \text{ is not greater than } n).$$

(a) How many different groups of 2 letters can be selected from the four letters  $\{A, B, C, D\}$ ? First, calculate the number (it is a small number) using the formula above by hand and then find which are these groups of 2 letters.

(b) How many groups of 4 people could be selected from a group of 20 competitors to go on a trip? Set up your formula and use your calculator to compute the number (this is a bigger number).

(c) What group size among 20 people will result in the largest number of possibilities? How many possibilities are there for this group size? Please, explain your answer with a short precise sentence recounting what you did.

(**Hint:** Here you have  $n = 20$  fixed, but  $k$  is allowed to vary; which  $k$  between 0 and 20 results in the largest  $C$ ?)