Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Consider the function $f(x) = \begin{cases} \frac{2x^2}{x-2}, & \text{if } x < 1 \\ \frac{3-x}{x^2-2}, & \text{if } 1 < x < \sqrt{2} \\ x, & \text{if } x \geq \sqrt{2} \end{cases}$

   (a) Find $\lim_{x \to 1} f(x)$. Show all work.

   (b) Is f continuous at $x = 1$? Explain.

2. (a) Write precisely the statement of the Intermediate Value Theorem. (It says that if two conditions hold for a function f, then some conclusion may be drawn about values of f.)

   (b) Apply the Intermediate Value Theorem to show that the equation $x^4 = 3 - x$ has a real root in the open interval $(1, 2)$. Please, show all steps.

3. Compute the following limits, showing **all steps**:

(a) $\lim_{x \to 1+} \frac{x^2-9}{x^2+2x-3}$

(b) $\lim_{x \to +\infty} \left(\sqrt{4x^2 + 1} - x \right)$

4. Use the definition of the derivative of a function f at a point a to compute the derivative $f'(2)$ if $f(x) = \sqrt{5 - x^2}$.

5. Use your differentiation rules to compute the derivative $f'(x)$ of the given function:

(a) $f(x) = x^4 - 3x^3 + 16x^2 - 7$

(b) $f(x) = 2\cos x - 3\sin x$

(c) $f(x) = (x-2)(2x+3)$

(d) $f(x) = \frac{x^2 - 2\sqrt{x}}{x}$