EXAM 4 - MATH 151 Friday, December 1

YOUR NAME:______________________ George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Recall the method for finding absolute extrema in a **closed** interval. Use the method for finding the absolute maximum and the absolute minimum of $f(x) = x^3 - 6x^2 + 9x + 2$ in the closed interval $[-1, 4]$.

2. (a) Use the Intermediate Value Theorem to show that the equation $2x - 1 - \sin x = 0$ has at least one real root. Briefly explain.

   (b) Use Rolle's Theorem to show that the equation $2x - 1 - \sin x = 0$ has at most one real root. Briefly explain.

   (c) What can you conclude from the previous two parts about the number of real solutions of the equation $2x - 1 - \sin x = 0$?

3. Use the sign table of the first derivative to find the intervals where the function $f(x) = x^4 + 8x^3 + 200$ is increasing or decreasing and all its local maximum and local minimum values.

4. Make a rough sketch of the graph of $f(x) = 1 + \sin x$ in the interval $[0, \frac{3\pi}{2}]$. Then find the area of the region under the curve $y = f(x)$ between $x = 0$ and $x = \frac{3\pi}{2}$.

5. This problem will guide you through the steps needed to plot by hand the graph of the function $f(x) = e^{2x} - e^x$. Please, follow instructions precisely and show your full work.

   (a) Find the domain Dom(f).

   (b) Find the x- and the y- intercepts of $y = f(x)$.

   (c) Compute

   $$\lim_{x \to -\infty} f(x) =$$

   $$\lim_{x \to +\infty} f(x) =$$

   (d) Compute $f'(x)$ and find the critical points.

   (e) Compute $f''(x)$ and find the critical points.

(f) Using the previous two parts make a sign table for f' and f'' and draw conclusions about the monotonicity and the concavity of f together with its local extrema and its inflection points. Show all pertinent information in the last line of your table (referring to f).

(g) Use your table and all previously gathered information to roughly plot the graph of $y = f(x)$. You **MUST** label your axes at the points of interest.