EXAM 1 - MATH 111

YOUR NAME:_____

Friday, September 14

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. The number N of deer present at time t (measured in years since the herd was introduced) on a certain wildlife reserve has been determined by ecologists to be given by the function

$$N(t) = \frac{12.36}{0.03 + 0.55^t}.$$

(a) How many deer were initially on the reserve?

(b) Calculate $N(10)$ and explain in a short and precise sentence the meaning of the number that you calculated.

(c) How much increase in the deer population do you expect from the 10th to the 15th year?

(d) What is the average rate of increase of the dear population between the 3rd and the 8th year?

2. Suppose a piece of aluminum is removed from an oven and is left to cool. The following table gives the temperature $A = A(t)$ in degrees Fahrenheit of the aluminum t minutes after it is removed from the oven.

$t =$ Minutes	0	30	60	90	120	150	180	210
$A =$ Temperature	302	152	100	81	75	73	72	72

(a) Explain the meaning of $A(75)$ and estimate its value.

(b) What is the average rate of change per minute of the temperature during the first half-hour of cooling?

(c) What is the average rate of change per minute of the temperature during the first half of the second hour of cooling?

(d) Use functional notation to express the temperature of the aluminum after 1 hour and 13 minutes. Estimate the temperature at that time.

(e) What is the temperature of the oven and what is the room temperature? Please, explain your answer with short precise phrases.

3. Consider the function
$$f(x) = 3(x - 2)^2 e^{-x}$$
in the interval $[1, 7]$. Our goal is to study in detail the graph $y = f(x)$. Please, follow the directions given closely and answer the following questions precisely.

(a) Graph $y = f(x)$ with your calculators setting your window for x between 1 and 7 and for y between -0.2 and 1.2. Reproduce your graph **carefully** here. If the shape is not very clear, you might want to zoom in and out to explore some details before drawing.

(b) What is the initial value for f in $[1, 7]$? If the graph is allowed to evolve further to the right outside $[1, 7]$ does it have a limiting value?

(c) Find the intervals of monotonicity (intervals where graph is increasing/decreasing) and the local extrema in $[1, 7]$. Summarize your results in a small table.

(d) Find the intervals of concavity (intervals where graph is concave up/concave down) and the inflection points in $[1, 7]$. Summarize your results in a small table.

4. Suppose that a manufacturer of widgets has fixed costs of $9,000 per month and that the variable cost is $ 15 per widget. Suppose, moreover, that the manufacturer sells the widgets for $25 each.

(a) Find a formula expressing the total cost C of this manufacturer in a month as a function of the number N of widgets produced in a month. State the units used.

(b) Express using functional notation the total cost if there are 250 widgets produced in a month and then calculate its value.

(c) Use a formula to express the manufacturer's total revenue R in a month as a function of the number N of widgets produced in a month. State the units used.

(d) Use a formula to express the profit P of the manufacturer as a function of the number N of widgets produced in a month.

(e) Express using functional notation the profit of the manufacturer if there are 250 widgets produced in a month and then calculate its value.

(f) How many widgets does the manufacturer need to produce to break even?