Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by

$$T(x_1, x_2) = (x_1 - 2x_2, -x_1 + 3x_2, 3x_1 - 2x_2).$$

   (a) Is T one-to-one? Please, explain.

   (b) Is T onto $\mathbb{R}^3$? Please, explain.

2. Suppose that $A = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$ and $AB = \begin{bmatrix} -1 & 2 & -1 \\ 6 & -9 & 3 \end{bmatrix}$. Find the matrix B using a method of your choice.

3. Consider the system of linear equations $\begin{cases} 8x_1 + 6x_2 & = & 2 \\ 5x_1 + 4x_2 & = & -1 \end{cases}$.

(a) Solve the system using the inverse matrix method.

(b) Solve the system using Cramer's Rule.

4. (a) Use the method of row reduction to compute the determinant

$$\begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ -1 & 2 & 8 & 5 \\ 3 & -1 & -2 & 3 \end{vmatrix} =$$

(b) Suppose that $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$. Then compute the determinant

$$\begin{vmatrix} 2d - a & 2e - b & 2f - c \\ g & h & i \\ a - g & b - h & c - i \end{vmatrix} =$$

5. Find the area of the triangle with vertices $(-2, -3)$, $(0, 5)$ and $(11, 2)$.