EXAM 3 - MATH 305 Friday, November 17

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Decide whether the given set H forms a subspace of the given space V. If yes prove. If not, give a counterexample.

   (a) $H = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} : x_1 - x_2 = x_2 - x_3 \right\}$ and $V = \mathbb{R}^4$.

   (b) $H = \{f \in F(\mathbb{R}) : f(0) = f(1)\}$ and $V = F(\mathbb{R})$ (set of all functions with domain $\mathbb{R}$).

2. (a) Show that $\{e^t, \sin t, \cos t\}$ is linearly independent in $F(\mathbb{R})$.

(b) Decide whether $\{1+t, t+t^2, t^2+t^3\}$ is linearly independent in $\mathbb{P}_3$.

(c) Does the set in Part (b) form a basis of $\mathbb{P}_3$? Explain.

3. Consider $A = \begin{bmatrix} 1 & -3 & 4 & -1 & 9 \\ -2 & 6 & -6 & -1 & -10 \\ -3 & 9 & -6 & -6 & -3 \\ 3 & -9 & 4 & 9 & 0 \end{bmatrix}$.

(a) Find a basis for $\mathsf{Nul}A$ and the nullity of A.

(b) Find a basis for $\mathsf{Col}A$ and the rank of A.

4. Find the rank of the matrix $A = \begin{bmatrix} 1 & -1 & 2-a \\ 2 & 1+a & 1 \\ -1 & 4 & 3+a \end{bmatrix}$ for all possible values of $a \in \mathbb{R}$.

Note: This matrix may have different ranks depending on which value a assumes and you are asked to explore all possibilities.

5. Consider two bases $\mathcal{B}$ and $\mathcal{C}$ of the vector space $\mathbb{P}_2$. If

$$\mathcal{C} = \{-2 + 2t + 3t^2, -8 + 5t + 2t^2, -7 + 2t + 6t^2\}$$

and $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P} = \begin{bmatrix} 1 & 2 & -1 \\ -3 & -5 & 0 \\ 4 & 6 & 1 \end{bmatrix}$, find $\mathcal{B}$.