Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Solve the initial value problem

$$\frac{dy}{dx} + 2xy = x, \quad y(1) = \frac{1}{2}.$$

2. Sarah won $1M in a lottery. She puts her winnings into a fund that has 5% annual return rate (compounded continuously), but each year she withdraws $ 20,000 for living expenses. How much money will be in the fund t years after its opening?

3. A 1500 gallon tank contains 1000 gallons of water with 10 lbs of salt dissolved in it. Water enters the tank at the rate of 10 gallons per hour having a salt concentration of $\frac{1}{10}\cos t$ pounds per gallon. If the mixture leaves the tank at the rate of 10 gallons per hour, how much salt would the tank contain at time t?

4. Solve the initial value problem

$$y'' - y' - 2y = 0, \quad y(0) = 1, \quad y'(0) = 2.$$

5. Consider the differential equation $(3e^x y + x) + e^x \dfrac{dy}{dx} = 0$.

   (a) Check whether it is exact.

   (b) Use an appropriate method to solve it.