EXAM 3 - MATH 310

Friday, November 17

George Voutsadakis

YOUR NAME:_____

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Find the Laplace transform $F(s)$, $s > 7$, of the function $f(t) = te^{7t}$ from "scratch" (that is without using tables, using only the definition of the Laplace transform).

2. Find the Laplace transform of the function $f(t) = \begin{cases} 0, & \text{if } t < 2 \\ t - 3, & \text{if } 2 \le t < 3 \\ -1, & \text{if } 3 \le t. \end{cases}$

3. Use Laplace transforms to solve the initial value problem

$$y'' - 2y' + 4y = 0, \quad y(0) = 2, \quad y'(0) = 0.$$

4. Find the solution of the initial value problem

$$y'' + 2y' + 2y = h(t), \quad y(0) = 0, \quad y'(0) = 0, \quad h(t) = \begin{cases} 0, & \text{if } 0 \le t < \pi \text{ or } t \ge 2\pi \\ 1, & \text{if } \pi \le t < 2\pi. \end{cases}$$

5. (a) Find the Laplace transform of

$$f(t) = \int_0^t e^{-\frac{\pi}{6}\tau} \sin(t - \tau) \cos \tau d\tau.$$

(b) Express the inverse Laplace transform of

$$F(s) = \frac{1}{(s+1)^3(s^2 + 16)}$$

in the form of an integral.