EXAM 4 - MATH 310

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YOUR NAME:_____

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Let $A = \begin{pmatrix} 3-2i & 1+i \\ 2-i & -2+3i \end{pmatrix}$ $x = \begin{pmatrix} 2 \\ i \\ 2+i \end{pmatrix}$, $y = \begin{pmatrix} 1-i \\ 2+i \\ 3 \end{pmatrix}$. Find the following:

(a) $A^T =$

(b) $\overline{A} =$

(c) $A^* =$

(d) $(x, y) =$

(e) $\|y\| =$

2. Solve the system $Ax = b$ for x, where $A = \begin{pmatrix} 1 & 0 & -1 \\ 3 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}$ and $b = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$.

3. Decide whether the vector $x = \begin{pmatrix} 6 \\ -8 \\ -1 \end{pmatrix} e^{-t} + 2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{2t}$ satisfies the differential equation

$$x' = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} x.$$ Show all details.

4. Find all eigenvalues and all eigenvectors of the matrix $A = \begin{pmatrix} -3 & \frac{3}{4} \\ -5 & 1 \end{pmatrix}$.

5. Solve the initial value problem

$$x' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} -1 \\ 3 \end{pmatrix}.$$