

YOUR NAME: \_\_\_\_\_

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Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Use Laplace transforms to solve the initial value problem

$$y'' + 2y' + y = 4e^{-t}, \quad y(0) = 0, \quad y'(0) = 1.$$

2. (a) Rewrite the following as a non-piece-wise defined function:

$$f(t) = \begin{cases} 3, & \text{if } 0 \leq t < 1, \\ -t^2 + 4, & \text{if } 1 \leq t < 2, \\ 0, & \text{if } t \geq 2. \end{cases}$$

- (b) Rewrite the following formula without using  $u$ 's.

$$g(t) = (t - 7)u_2(t) - (t - 11)u_5(t).$$

- (c) Find the Laplace transform of the function  $g$  of Part (b).

3. Suppose that you are given  $\mathcal{L}\{f(t)\} = F(s)$ . Compute from scratch (without consulting tables or using formulas other than the definition of the Laplace transform) the Laplace transform of  $f(2t)$ .