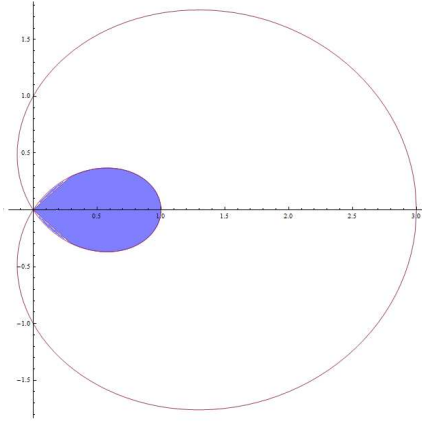


YOUR NAME: _____

George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Find the area of the region enclosed by the inner loop of the limaçon $r = 2 \cos \theta - 1$.



2. (a) An ellipse has vertices at $(-9, 5)$ and $(-7, 5)$ and foci at $(-8, 7)$ and $(-8, 3)$. Find an equation for the ellipse.

- (b) A hyperbola has equation $\frac{(y - 7)^2}{9} - \frac{(x - 2)^2}{16} = 1$. Find the exact locations of its foci.

3. (a) Give an equation for the sphere with center at $(3, -5, 7)$ and radius $R = 4$.
- (b) Give a system of parametric equations for the straight line segment joining $P = (3, -5, 7)$ to $Q = (1, 2, 0)$ traversed in the direction from P to Q .
4. (a) Find the point of intersection of the straight line $\mathbf{r}(t) = \langle 0, 0, 1 \rangle + t\langle 3, -1, -2 \rangle$ with the plane $z = -3$.

- (b) Check whether the straight lines $\begin{cases} x = 3t \\ y = 1 - t \\ z = 1 - 2t \end{cases}$ and $\begin{cases} x = 3 + 2t \\ y = -1 + t \\ z = 5 - 3t \end{cases}$ are intersecting lines.

5. (a) Consider the vectors \overrightarrow{PQ} and \overrightarrow{PR} , where $P = (0, 0, 1)$, $Q = (2, 1, 0)$ and $R = (-1, 5, 1)$. Decide whether the angle between these vectors is acute, right or obtuse. Show all steps and state clearly your conclusion.

- (b) Consider the vectors $\mathbf{u} = \langle 4, -1, 5 \rangle$ and $\mathbf{v} = \langle 2, 1, 1 \rangle$. Find the decomposition $\mathbf{u} = \mathbf{u}_{\parallel} + \mathbf{u}_{\perp}$ of \mathbf{u} with respect to \mathbf{v} .