EXAM 2 - MATH 251 Friday, October 12

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. (a) Find the values of the parameters a and b so that the plane with equation $ax+3y-z=b$ is parallel to the line $\boldsymbol{r}(t)=\langle 1+2t,-3t,7-5t\rangle$ and passes through the point $(-10,10,7)$.

   (b) Suppose that $\boldsymbol{r}(t)=\langle t^2,1-t,4t\rangle$, $\boldsymbol{s}(2)=\langle 1,3,3\rangle$ and $\boldsymbol{s}'(2)=\langle -1,4,1\rangle$. Find the derivative of $\boldsymbol{r}(t)\cdot\boldsymbol{s}(t)$ at $t=2$.

   (c) Suppose $\boldsymbol{r}(t)=\langle t^2,2t,9t^{-2}\rangle$, $g(4)=3$ and $g'(4)=-9$. Evaluate $\frac{d}{ds}\boldsymbol{r}(g(s))\mid_{s=4}$.

2. Find a parametrization of the tangent line to $\boldsymbol{r} = \langle 1 - t^2, 5t, 2t^3 \rangle$ at $t = 2$.

3. Consider the curve $\boldsymbol{r}(t) = \langle \sin 3t, \cos 3t, 4t \rangle$.

   (a) Find the length $s(t)$ of $\boldsymbol{r}(t)$ between $t = 0$ and an arbitrary time t.

   (b) Give an arc length parametrization of $\boldsymbol{r}(t)$.

4. Evaluate the curvature of

$$\boldsymbol{r}(t) = \langle 3 - t, e^{2t}, t - t^2 \rangle$$

at $t = 1$.

5. Find the unit tangent and the unit normal vector to $\boldsymbol{r}(t) = \langle \ln t, 2t, t^2 \rangle$ at $t = 1$.