EXAM 3 - MATH 251 Friday, November 9

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Consider the function $f(x, y, z) = \frac{xy^3}{\sqrt{z}}$.

   (a) Find the linearization $L(x, y, z)$ of f at $(x, y, z) = (3, 1, 4)$.

   (b) Compute the differential df of f.

   (c) Use the differential you found on Part (b) to approximate the value of the function $f(x, y, z)$ at the point $(x, y, z) = (3.01, 0.98, 4.04)$.

2. (a) A robotic thermometer on the plane is following the trajectory $\boldsymbol{c}(t) = (t^2, t)$. The temperature varies according to the formula $T(x, y) = x^2 \cos y$. Find how fast is the thermometer-registered temperature changing at $t = 2$.

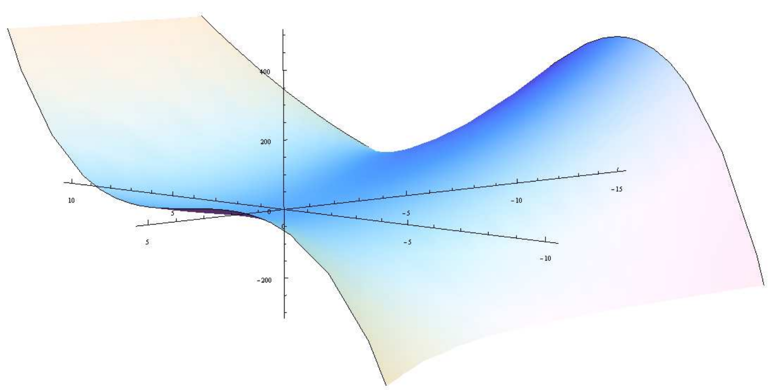
(b) Suppose the temperature in space varies according to $T(x, y, z) = xe^{y-z}$. A bug is located at the point $(3, 9, 4)$.

(i) If the bug starts walking towards the point $(5, 7, 3)$ following a straight line at unit speed, how fast is the bug's temperature changing?

(ii) If the bug is temperature sensitive, i.e., it is programmed to veer towards maximum temperature changes, in which direction will that bug start walking starting from the same initial position?

3. Find the partial derivative $\dfrac{\partial h}{\partial q}$ at $(q, r) = (3, 2)$ if $h(x, y) = xe^y$, $x = q^3$ and $y = qr^2$.

4. Determine the critical points of $f(x, y) = x^3 + x^2y + 2y^2$ and identify whether they give rise to local min/max or saddle points.



5. Compute $\iint_{\mathcal{D}} \cos(2x+y)\,dA$, where $\mathcal{D}$ is the region shown below. (Make sure to express your region formally, as done in class, before starting integration.)