EXAM 1 - CSCI 341 Friday, September 21

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. (a) Show that $A \to \neg B \equiv \neg(A \wedge B)$

   (b) The contrapositive of $A \to B$ is

   (c) The negation of the statement "George is a Democrat or George is a Republican" is the statement

   (d) The converse of the statement "If there are more than 50 Republican Senators, then there are at most 49 Democratic Senators" is the statement

   (e) The statement "If $6 \mid x$ then $4 \mid x$" is _________
   Proof:

   (f) Prove the statement "If $x^2 - 2x + 3$ is odd, then x is even", where x is assumed to be an integer.

2. Fill in the blanks:

   (a) $\{a, b\} \quad \{a, \{b\}, \{a, b\}\}$;

   (b) $\{0, 1, 2, 3, 4, 5\} \cap (\{0, 3, 4, 5, 6, 7\} - \{0, 1, 2, 3\}) =$

   (c) $\mathcal{P}(\{0, a\}) =$

   (d) $A - B = \{ \qquad : \qquad \}$;

   (e) $A \oplus B = \{ \qquad : \qquad \}$;

   (f) Let $A = \{2k + 7 : k \in \mathbb{Z}\}$ and $B = \{4k + 3 : k \in \mathbb{Z}\}$. Show that $B \subset A$.
   Proof:

3. Fill in the blanks:

(a) $A \times B = \{ \qquad\qquad : \qquad\qquad\qquad \};$

(b) $\mathsf{cons}(\mathsf{head}(\langle\langle a\rangle, \langle\rangle, \langle a, b\rangle\rangle), \mathsf{tail}(\mathsf{tail}(\langle\langle a\rangle, \langle a, b\rangle, \langle c\rangle, \langle\langle\rangle, \langle a, b\rangle\rangle\rangle))) =$

(c) $\{\Lambda, abab, aabbabab, aaabbbababab, aaaabbbbabababab, \ldots\} =$

(d) Only in (d), assume $L = \{\Lambda, a, bab\}$ and $M = \{aba, b, bab\}$.

$LM =$

(e) $L^+ =$

(f) The statement that for all languages L and M,

$$L^* - M^* = (L - M)^*$$

is ________

Proof:

4. Fill in the blanks:

(a) If $f : A \to B$ and $S \subseteq A$,

$f(S) = \{ \qquad : \qquad \};$

(b) If $f : A \to B$ and $T \subseteq B$,

$f^{-1}(T) = \{ \qquad : \qquad \};$

(c) Finish the formal statement of the division algorithm:

For every integers m and n, with $n \neq 0$,

(d) Apply Euclid's algorithm to find the gcd of 612 and 50. Show carefully all iterations of the algorithm:

(e) $\mathsf{dist}(0, \mathsf{map}(+)(\mathsf{pairs}(\mathsf{seq}(3), \mathsf{seq}(3)))) =$

(f) The statement that, for every function $f : A \to B$ and every subset $G \subseteq B$,

$$f(f^{-1}(G)) = G$$

is ________

Proof:

5. (a) A function $f : A \to B$ is injective (or 1-1) if

(b) A function $f : A \to B$ is surjective (or onto) if

(c) Consider the function $f : \mathbb{Z}_6 \to \mathbb{Z}_6$, defined by $f(x) = 5x \mod 6$.

x	$f(x)$

(i) The statement "f is injective" is _______, because

(ii) The statement "f is surjective" is _______, because

(iii) The statement "f has an inverse" is _______, because

(d) Let $f : A \to B$ and $g : B \to C$ be functions. The statement "If $g \circ f$ is surjective, then g is surjective" is _________

Proof: