EXAM 1 - CSCI 341 YOUR NAME:

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. (a) Show that $A \to \neg B \equiv \neg (A \land B)$

- (b) The contrapositive of $A \to B$ is
- (c) The negation of the statement "George is a Democrat or George is a Republican" is the statement
- (d) The converse of the statement "If there are more than 50 Republican Senators, then there are at most 49 Democratic Senators" is the statement
- (e) The statement "If $6 \mid x$ then $4 \mid x$ " is _____ **Proof:**
- (f) Prove the statement "If $x^2 2x + 3$ is odd, then x is even", where x is assumed to be an integer.

2. Fill in the blanks:

- (a) $\{a,b\} \ \{a,\{b\},\{a,b\}\};$
- (b) $\{0, 1, 2, 3, 4, 5\} \cap (\{0, 3, 4, 5, 6, 7\} \{0, 1, 2, 3\}) =$

(c) $\mathcal{P}(\{0,a\}) =$

- (d) $A B = \{$: };
- (e) $A \oplus B = \{$: };
- (f) Let $A = \{2k + 7 : k \in \mathbb{Z}\}$ and $B = \{4k + 3 : k \in \mathbb{Z}\}$. Show that $B \subset A$. **Proof**:

3. Fill in the blanks:

- (a) $A \times B = \{$: };
- $(b) \ \operatorname{cons}(\operatorname{head}(\langle\langle a\rangle,\langle\langle a,b\rangle\rangle\rangle),\operatorname{tail}(\operatorname{tail}(\langle\langle a\rangle,\langle a,b\rangle,\langle c\rangle,\langle\langle a,b\rangle\rangle\rangle))) =$
- (d) Only in (d), assume $L = \{\Lambda, a, bab\}$ and $M = \{aba, b, bab\}$. LM =
- (e) $L^+ =$
- (f) The statement that for all languages L and M,

$$L^* - M^* = (L - M)^*$$

is _____ Proof:

4. Fill in the blanks:

(a) If $f: A \to B$ and $S \subseteq A$,

 $f(S) = \{ : \};$

(b) If $f: A \to B$ and $T \subseteq B$,

 $f^{-1}(T) = \{ : \};$

(c) Finish the formal statement of the division algorithm:

For every integers m and n, with $n \neq 0$,

(d) Apply Euclid's algorithm to find the gcd of 612 and 50. Show carefully all iterations of the algorithm:

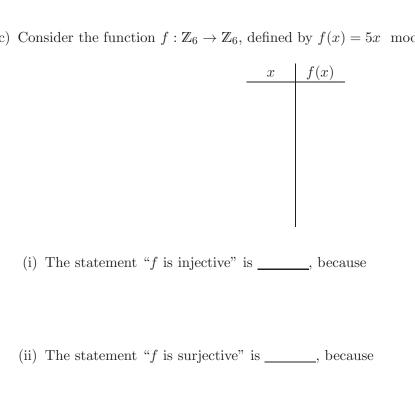
(e) dist(0, map(+)(pairs(seq(3), seq(3)))) =

(f) The statement that, for every function $f: A \to B$ and every subset $G \subseteq B$,

$$f(f^{-1}(G)) = G$$

is _____ Proof:

- 5. (a) A function $f: A \to B$ is injective (or 1-1) if
 - (b) A function $f: A \to B$ is surjective (or onto) if
 - (c) Consider the function $f : \mathbb{Z}_6 \to \mathbb{Z}_6$, defined by $f(x) = 5x \mod 6$.



(iii) The statement "f has an inverse" is _____, because

(d) Let $f: A \to B$ and $g: B \to C$ be functions. The statement "If $g \circ f$ is surjective, then g is surjective" is _____ **Proof:**