Read each problem very carefully before starting to solve it. Each problem is worth 10 points. It is necessary to show all your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. (a) Give the formal inductive definition of the set of regular languages over alphabet $A$.
(b) Give a regular expression for the language of all strings over alphabet $A=\{a, b, c\}$ that start with $a b c$ or end with $c b a$.
(c) Give a regular expression for the language of all strings over the alphabet $A=\{0,1\}$ that do not end with 01.
(d) Give a formal recursive definition of the operator $L$ that takes as input a regular expression over alphabet $A$ and outputs the regular language that is represented by the given regular expression.
(e) The statement "The language $L$ of all strings over alphabet $A=\{a, b\}$ that have odd length is regular" is $\qquad$ Proof:
2. (a) The transition function of a DFA is a function $\delta$ : $\qquad$ $\rightarrow$ $\qquad$ , where
(b) The transition function of an NFA is a function $\delta$ : $\qquad$ $\rightarrow$ $\qquad$ , where
(c) An NFA $N$ accepts a string $w$ over its input alphabet $A$ if, by definition
(d) Give an NFA that recognizes the regular language $\{a, b\}^{*}\{a b\}$.
(e) Apply the NFA to DFA algorithm to obtain the DFA that corresponds to the NFA you constructed in Part (d).
3. (a) Give the form of productions that are allowed in a regular grammar, explaining your symbols.
(b) Under each regular expression provide the regular grammar whose language is the same as that represented by the corresponding regular expression:
$\qquad$
(c) Find a regular grammar for the language represented by the regular expression $a^{*} b c^{*}+a c$.
(d) Apply the algorithm converting a regular grammar to an NFA to obtain an NFA corresponding to the regular grammar that you gave in Part (c).
4. (a) The PDA instruction $\langle A, a, X, \operatorname{push}(Y), B\rangle$ can be shown pictorially as follows:

The interpretation of it is that:
(b) Give the formal definition of an instantaneous description (ID) in a computation of a PDA.
(c) Consider the following PDA with initial stack symbol $X$. Give a formal description of an accepting computation on input string 110011 in terms of IDs.

(d) "The language accepted by the PDA of Part (c) is the language of palindromes over $A=\{0,1\}$ " is $\qquad$ proof:
(e) Give a PDA that recognizes the language $\left\{a^{n+1} b^{n} a: n \geq 1\right\}$ (make sure to state which mode of acceptance your PDA is using).
5. (a) Give an example of a language that is context-free but not regular (write formally).
(b) Apply the relevant algorithm to construct a PDA that accepts the context-free language described by

$$
\begin{aligned}
& S \rightarrow A B A \\
& A \rightarrow a A \mid \Lambda \\
& B \rightarrow b B \mid \Lambda .
\end{aligned}
$$

(c) Apply the relevant algorithm to eliminate $\Lambda$-productions from the following grammar:

$$
\begin{aligned}
& S \rightarrow a S A|b s B| A \\
& A \rightarrow a B b \mid b B a \\
& B \rightarrow a B|b B| \Lambda
\end{aligned}
$$

(d) Apply the relevant algorithm to produce the Chomsky Normal Form of the following context-free grammar.

$$
\begin{aligned}
& S \rightarrow C \\
& C \rightarrow a C a|b C b| A \\
& A \rightarrow a B b \mid b B a \\
& B \rightarrow a B|b B| b
\end{aligned}
$$

