EXAM 4 - CSCI 341 Friday, November 30

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. (a) Give the definition of a **context-free language**.

   (b) True or false?

   (i) $L = \{a^n b^n c^n : n \in \mathbb{N}\}$ is context free. _____

   (ii) $M = \{a^n b^n c^k : n, k \in \mathbb{N}\}$ is context-free. _____
   Proof of (ii):

   (c) The union of two context free languages is context-free. _____
   Proof:

   (d) The intersection of two context-free languages is context-free. _____
   Proof:

   (e) The complement of a context-free language is context-free. _____
   Proof:

2. (a) Give a list of the six components in the formal definition of a **Turing Machine** and a short description of each.
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   (b) Give the "official form" of a **Turing Machine Instruction** with a short explanation of its meaning.

   (c) Design a Turing Machine that on input $w \in \{0,1\}^*$, it shifts the contents of its tape one cell to the left and stops (there is no prescription on where the head must be positioned).

   (d) Design a Turing Machine that on input $w \in \{a,b\}^*$ changes every second "a" in the input to a "b" and halts with the head positioned on the first symbol of the output.

3. (a) Describe what is an **effective enumeration** of all Turing Machines.

(b) (i) Describe what is means for a decision problem to be **decidable**.

(ii) Describe what it means for a decision problem to be **partially decidable**.

(c) Define formally the Halting Problem describing carefully the input instances and the question posed.

(d) Define formally the Total Problem describing carefully the input instances and the question posed.

(e) True or False?

(i) The Halting Problem is partially decidable. _____

(ii) The Halting Problem is decidable. _____

(iii) There is an effective enumeration of all total computable functions. _____

(iv) The Total Problem is decidable. _____

(v) The complement of the Halting Problem is partially decidable. _____

4. (a) Define a **nondeterministic algorithm**.

(b) Define the class P.

(c) Define the class NP.

(d) Define formally the decision version of the Traveling Salesman Problem describing carefully the input instances and the question posed.

(e) Define formally the Clique Problem describing carefully the input instances and the question posed.

(f) The statement "The Clique Problem is in NP" is _____
Proof:

5. (a) Define formally a **quantified Boolean formula**.

(b) Define recursively the function val from the set of quantified Boolean formulae to the set $\{T, F\}$.
Basis:

Recursion Cases:

(c) Construct the Recursion Tree to decide the truth value of the quantified Boolean formula $\exists x \forall y((\neg x \vee y) \wedge (x \vee \neg y))$

(d) Define formally the Quantified Boolean Formula Problem describing carefully the input instances and the question posed.