Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. In this quiz, you are asked to design three Turing Machines of increasing complexity.

   (a) Design a Turing Machine M_1 that does the following: On input $x_0 x_1 \cdots x_n \in \{a, b, c\}^*$ it replaces all occurrences of c by a and halts with its head scanning the first symbol of the output.
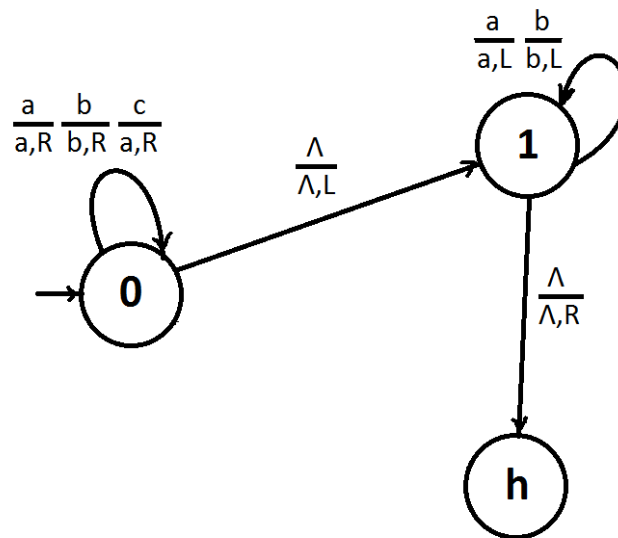
   Turing Machine M_1: Input string $w \in \{a, b, c\}^*$.
   Scan to the right until Λ.
       If symbol is c replace by a.
   Scan to the left until Λ.
   Move right.

(b) Design a Turing Machine M_2 that does the following: On input $x_0 x_1 \cdots x_{n-1} x_n \in \{a, b\}^*$, it leaves on its tape the output $x_n x_0 x_1 \cdots x_{n-1}$ and halts with its head scanning x_n.

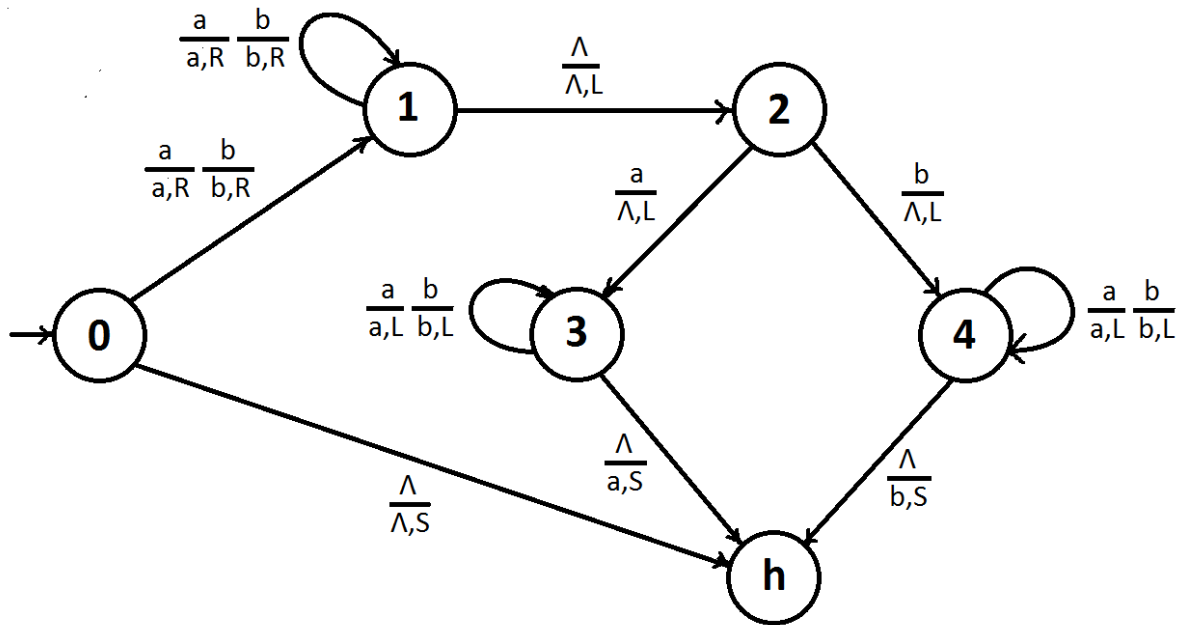Turing Machine M_2: Input string $w \in \{a, b\}^*$.
If input string is Λ stop and accept.
Else, scan to the right until Λ.
      Go one left, scan x_n, remember x_n and erase x_n.
      Scan to the left until Λ.
      Replace Λ by x_n and halt.

(c) Design a Turing Machine M_3 that on input $x_0 x_1 \cdots x_n \in \{(,)\}^*$ balances left and right parentheses, i.e., halts accepting if the input expression of parentheses is syntactically sound (e.g. "(()())") and rejects otherwise (e.g. ")" or "(()(()").

Turing Machine M_3: Input string $w \in \{(,)\}^*$.

If input string is Λ stop and accept.

Else, repeat until no) is found:

  Scan to the right until) or Λ.

  If) is found replace by X.

  Scan to the left until (.

  Replace by X.

Scan to the left over all X's until Λ (check that no (are left).

Stop and accept.