QUIZ 1 - SOLUTIONS

Friday, August 31

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1. (a) Consider propositional variables A, B and C. Use the truth table method to show that the two propositions $A \to (B \vee C)$ and $(A \wedge \neg B) \to C$ are equivalent.

   Construct the truth tables for $A \to (B \vee C)$ and $(A \wedge \neg B) \to C$ and show that the corresponding columns are identical:

A	B	C	$B \vee C$	$A \to (B \vee C)$	$\neg B$	$A \wedge \neg B$	$(A \wedge \neg B) \to C$
T	T	T	T	T	F	F	T
T	T	F	T	T	F	F	T
T	F	T	T	T	T	T	T
T	F	F	F	F	T	T	F
F	T	T	T	T	F	F	T
F	T	F	T	T	F	F	T
F	F	T	T	T	T	F	T
F	F	F	F	T	T	F	T

   (b) Assume the following:

   A : "n is a nonnegative integer"
   B : "n is even"
   C : "n is odd"

   Express in ordinary English the following propositions:

   - $A \to (B \vee C)$:

     If n is a nonnegative integer, then n is even or n is odd.

   - $(A \wedge \neg B) \to C$:

     If n is a nonnegative integer and n is not even, then n is odd.

2. (a) Let A be a propositional variable and let F be a false proposition. Construct the truth table for $(\neg A \to F) \to A$.

A	F	$\neg A$	$\neg A \to F$	$(\neg A \to F) \to A$
T	F	F	T	T
F	F	T	F	T

In class, we gave a proof of the statement "There are infinitely many primes". Answer the following questions concerning the proof. You must be concise and precise.

[Here is a reminder of the proof:

> Suppose there exist only finitely many primes, say $p_1, p_2, \ldots, p_n$. Consider the number
>
> $$k = p_1 p_2 \cdots p_n + 1.$$
>
> Since it is larger than all of $p_1, \ldots, p_n$, it cannot be a prime. By the Decomposition into Primes, it is a product of primes, say $k = p_{i_1} \cdots p_{i_\ell}$. Now we have
>
> $$p_1 p_2 \cdots p_n + 1 = p_{i_1} \cdots p_{i_\ell}.$$
>
> This gives $1 = p_{i_1} \cdots p_{i_\ell} - p_1 p_2 \cdots p_n$. But the right hand side is divisible by p_{i_1} (since it is a prime and, therefore, among the $p_1, \ldots, p_n$). Thus, $p_{i_1} \mid 1$, a contradiction.]

(b) We gave a proof by <u>contradiction</u>

(c) We assumed that <u>there were finitely many primes</u>

(d) Under (c) we proved that <u>a prime number divides 1</u>

(e) We concluded that <u>there are infinitely many primes</u>

(f) Which statement should A stand for in Part (a) so that the proof described in Parts (c)-(e) is mirrored by the proposition $(\neg A \to F) \to A$?

A : "There are infinitely many primes"

Then the proposition $(\neg A \to F) \to A$ gives the statement:

> If the existence of finitely many primes implies a false statement, then there are infinitely many primes.