QUIZ 2 - CSCI 341 Friday, September 7

George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Take a minute to think about the following statements regarding integers. State whether each of the given statements is true of false and prove its truth or falsity after you indicate the proof method you will be using.

   (a) "If x divides $3y$, then x divides y"

   This statement is False

   Proof by Counterexample

   Proof: Take $x = 3$ and $y = 2$. Then $x \mid (3 \cdot 2)$, but $x \nmid 2$.

   (b) "If $x^2 + 3$ is even, then x is odd"

   This statement is True

   Proof by Contraposition

   Proof: Suppose x is even. Then $x = 2k$ for some integer k. Thus

   $$x^2 + 3 = (2k)^2 + 3 = 4k^2 + 3 = 2(2k^2 + 1) + 1.$$

   Thus $x^2 + 3$ is odd.

2. Fill in the missing symbol or the missing set:

   (a) $\{a\} \in \{\{a\}, \{a, b\}\}$;

   (b) $\{a\} \subseteq \{a, b, c\}$;

   (c) $\{a, b, c\} \cup \{a, b\} = \{a, b, c\}$

   (d) $\{0, 1, 2, 3\} \cap \{4, 5\} = \emptyset$

   (e) $\{0, 1, 2, 3\} - \{0, 4, 5\} = \{1, 2, 3\}$

3. Prove that the following statement holds, for all sets A, B and C (please, show the "proof template keywords" clearly and explain concisely each step of your proof to facilitate my checking):

$$A - (B \cap C) \subseteq (A - B) \cup (A - C).$$

   Suppose $x \in A - (B \cap C)$.

   Then $x \in A$ and $x \notin B \cap C$.

   So $x \in A$ and not($x \in B$ and $x \in C$).

   So $x \in A$ and ($x \notin B$ or $x \notin C$) (by De Morgan's Laws).

   Equivalently, ($x \in A$ and $x \notin B$) or ($x \in A$ and $x \notin C$) (using Distributivity).

   So we conclude $x \in A - B$ or $x \in A - C$.

   This shows that $x \in (A - B) \cup (A - C)$.