QUIZ 3 - CSCI 341 Friday, September 14

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Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Fill in the missing information in formal notation (not in verbal descriptions):

   (a) $\{a, b, c\} \times \{A, B\} \times \{0, 1\} =$

   (b) $\mathsf{tail}(\langle\langle a, \langle\rangle\rangle, \langle a\rangle, \langle a, b\rangle\rangle) =$

   (c) $\mathsf{cons}(\langle a\rangle, \mathsf{tail}(\mathsf{tail}(\langle\langle a, \langle\rangle\rangle, \langle a\rangle, \langle a, b\rangle\rangle))) =$

   (d)
$$\begin{aligned} L &= \{aab, aabb, aabbb, \ldots\} \cup \{abab, ababb, ababbb, \ldots\} \\ &\quad \cup \{abbab, abbabb, abbabbb, \ldots\} \cup \cdots \\ &= \{ \qquad\qquad : \qquad\qquad\qquad \} \end{aligned}$$

2. (a) Let L be the language $L = \{\Lambda, ab\}$ over the alphabet $A = \{a, b\}$.

   (i) $L^3 =$

   (ii) $L^+ =$

   (b) Decide on whether the following statements are true or false and provide a proof (making sure to following a template closely):

   (i) $L(M \cap N) \subseteq LM \cap LN$, for all languages L, M, N over some alphabet A.

   This statement is ____________

   Proof:

   (ii) $LM \cap LN \subseteq L(M \cap N)$, for all languages L, M, N over some alphabet A.

   This statement is ____________

   Proof:

3. (a) Let $f : A \to B$ be a function. Define the following sets using formal notation:

   (i) If $S \subseteq A$, then

$$f(S) = \{ \qquad : \qquad \}$$

   (ii) If $T \subseteq B$, then

$$f^{-1}(T) = \{ \qquad : \qquad \}$$

(b) In this part f denotes the function $\mathsf{ceiling} = \lceil\ \rceil$. Find the following:

   (i) The type of f is

   (ii) $f([2.3, 7]) =$

   (iii) $f^{-1}(\{-5, -4\}) =$

(c) Consider the following statement:

For every function $f : A \to B$ and all subsets $E, F \subseteq A$,

$$f(E \cap F) = f(E) \cap f(F).$$

The statement above is ________

Proof: