QUIZ 3 SOLUTIONS - CSCI 341 — Friday, September 14

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Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Fill in the missing information in formal notation (not in verbal descriptions):

   (a)
$$\begin{aligned}\{a,b,c\}\times\{A,B\}\times\{0,1\} &= \{(a,A,0),(a,A,1),(a,B,0),(a,B,1),\\ &\quad (b,A,0),(b,A,1),(b,B,0),(b,B,1),\\ &\quad (c,A,0),(c,A,1),(c,B,0),(c,B,1)\};\end{aligned}$$

   (b) $\mathsf{tail}(\langle\langle a,\langle\rangle\rangle,\langle a\rangle,\langle a,b\rangle\rangle) = \langle\langle a\rangle,\langle a,b\rangle\rangle$;

   (c)
$$\begin{aligned}\mathsf{cons}(\langle a\rangle,\mathsf{tail}(\mathsf{tail}(\langle\langle a,\langle\rangle\rangle,\langle a\rangle,\langle a,b\rangle\rangle))) &= \mathsf{cons}(\langle a\rangle,\mathsf{tail}(\langle\langle a\rangle,\langle a,b\rangle\rangle))\\ &= \mathsf{cons}(\langle a\rangle,\langle\langle a,b\rangle\rangle)\\ &= \langle\langle a\rangle,\langle a,b\rangle\rangle.\end{aligned}$$

   (d)
$$\begin{aligned}L &= \{aab, aabb, aabbb,\ldots\}\cup\{abab, ababb, ababbb,\ldots\}\\ &\quad \cup\{abbab, abbabb, abbabbb,\ldots\}\cup\cdots\\ &= \{ab^mab^n : m\geq 0, n>0\}\end{aligned}$$

2. (a) Let L be the language $L=\{\Lambda, ab\}$ over the alphabet $A=\{a,b\}$.

   (i) $L^3 = \{\Lambda, ab, abab, ababab\}$

   (ii) $L^+ = \{(ab)^n : n\geq 0\}$

   (b) Decide on whether the following statements are true or false and provide a proof (making sure to following a template closely):

   (i) $L(M\cap N)\subseteq LM\cap LN$, for all languages L,M,N over some alphabet A.

   This statement is <u>True</u>

   Proof:
   Let $x\in L(M\cap N)$.
   Then $x=\ell y$, where $\ell\in L$, $y\in M\cap N$.
   So $x=\ell y$, where $\ell\in L$ and $y\in M$ and $y\in N$.
   Thus, $x\in LM$ and $x\in LN$.
   So $x\in LM\cap LN$.

   (ii) $LM\cap LN\subseteq L(M\cap N)$, for all languages L,M,N over some alphabet A.

   This statement is <u>False</u>

   Proof: Consider the languages L,M,N over $A=\{a\}$, defined as follows:

$$L=\{a,aa\},\qquad M=\{a\},\qquad N=\{\Lambda\}.$$

   Then, we can easily check that

$$\begin{aligned}&aa\in LM\cap LN;\\ &aa\notin L(M\cap N)=\emptyset.\end{aligned}$$

3. (a) Let $f : A \to B$ be a function. Define the following sets using formal notation:

(i) If $S \subseteq A$, then

$$f(S) = \{f(s) : s \in S\}$$

(ii) If $T \subseteq B$, then

$$f^{-1}(T) = \{a \in A : f(a) \in T\}$$

(b) In this part f denotes the function ceiling $= \lceil \ \rceil$. Find the following:

(i) The type of f is $\mathbb{R} \to \mathbb{Z}$

(ii) $f([2.3, 6.7]) = \{3, 4, 5, 6, 7\}$

(iii) $f^{-1}(\{-5, -4\}) = (-6, -4]$

(c) Consider the following statement:

For every function $f : A \to B$ and all subsets $E, F \subseteq A$,

$$f(E \cap F) = f(E) \cap f(F).$$

The statement above is <u>False</u>

Proof: Consider the function $f : \{a, b\} \to \{0, 1\}$, defined by $f(a) = f(b) = 0$.
Let $E = \{a\}$ and $F = \{b\}$.
Then we can easily check that:

$$\begin{array}{rcl} 0 & \in & f(E) \cap f(F) = f(\{a\}) \cap f(\{b\}); \\ 0 & \notin & f(E \cap F) = f(\emptyset). \end{array}$$