Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Fill in the missing information (please, try to be formal, precise and concise):

   (a) The sets A and B have **the same cardinality** if

   (b) A set A is defined to be **countably infinite** if

   (c) Cantor's Theorem: For any set A,

   (d) Cantor's Theorem is proved by

   (e) The same technique (as that named in (d)) is also used to show, e.g., that

2. Let $A = \{x \in \mathbb{N} : x \mod 7 = 5\}$. Prove (without skipping any details) that A is countably infinite.

3. Consider the alphabet A of all symbols allowed in Java® programs. Give the characteristics (not asking for exact numbers).

   (a) The cardinality of A is _____

   (b) The cardinality of A^* is _____ because

   (c) The cardinality of all valid (or correct) Java programs is _____, because

   (d) The cardinality of the set $\mathcal{P}(A^*)$ of all languages over A is _____ by

   (e) From (c) and (d) we can conclude that

       because