QUIZ 6 SOLUTIONS - CSCI 341 Friday, October 5

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Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

Please, recall (or take it as a definition if you have not seen it before) that the product of zero elements is defined as 1 and that the sum of zero elements is defined as 0, i.e., we have:

$$\sum \emptyset = 0 \quad \text{and} \quad \prod \emptyset = 1.$$

1. (a) Consider the function $\mathsf{Prod} : \mathsf{Lists}[\mathbb{R}] \to \mathbb{R}$, defined as follows:

$$\mathsf{Prod}(\langle x_1, x_2, \ldots, x_n \rangle) = x_1 \cdot x_2 \cdot \cdots \cdot x_n, \text{ for all } x_1, x_2, \ldots, x_n \in \mathbb{R}.$$

Give a recursive definition of Prod:

Do here some analysis if you need to:

$$\begin{aligned} \mathsf{Prod}(\langle x_1, x_2, \ldots, x_n \rangle) &= x_1 \cdot x_2 \cdot \cdots \cdot x_n \\ &= x_1 \cdot (x_2 \cdot \cdots \cdot x_n) \\ &= \mathsf{head}(\langle x_1, x_2, \ldots, x_n \rangle) \cdot \mathsf{Prod}(\mathsf{tail}(\langle x_1, x_2, \ldots, x_n \rangle)). \end{aligned}$$

Basis: $\mathsf{Prod}(\langle\rangle) = 1$;
Induction: $\mathsf{Prod}(L) = \mathsf{head}(L) \cdot \mathsf{Prod}(\mathsf{tail}(L))$, if $L \neq \langle\rangle$.

(b) In Part (b) you might use, if you decide it is convenient to do so, the function Prod that you defined in Part (a), in the spirit of incrementally building computer code by reusing previously defined functions and procedures.

Consider the following "special product" function $\mathsf{SpecProd} : \mathsf{Lists}[\mathbb{R}] \to \mathbb{R}$, defined as follows:

$$\mathsf{SpecProd}(\langle x_1, x_2, \ldots, x_n \rangle) = x_1^1 \cdot x_2^2 \cdot \cdots \cdot x_n^n, \text{ for all } x_1, x_2, \ldots, x_n \in \mathbb{R}.$$

Give a recursive definition of $\mathsf{SpecProd}$:

Do here some analysis if you need to:

$$\begin{aligned} \mathsf{SpecProd}(\langle x_1, x_2, \ldots, x_n \rangle) &= x_1^1 \cdot x_2^2 \cdot \cdots \cdot x_n^n \\ &= (x_1 \cdot x_2 \cdots \cdot x_n) \cdot (x_2^1 \cdot \cdots \cdot x_n^{n-1}) \\ &= \mathsf{Prod}(\langle x_1, x_2, \ldots, x_n \rangle) \cdot \mathsf{SpecProd}(\mathsf{tail}(\langle x_1, x_2, \ldots, x_n \rangle)). \end{aligned}$$

Basis: $\mathsf{SpecProd}(\langle\rangle) = 1$;
Induction: $\mathsf{SpecProd}(L) = \mathsf{Prod}(L) \cdot \mathsf{SpecProd}(\mathsf{tail}(L))$, if $L \neq \langle\rangle$.

2. This problem is given in your textbook in the "Challenges" section, but (I believe) we can all solve it successfully.

   Consider the function $f : \mathbb{N} \to \mathbb{N}$ defined by the following formula:

$$f(x) \;=\; \mathsf{if}\ x > 4\ \ \mathsf{then}\ x - 4 \quad \mathsf{else}\ f(f(x+5)).$$

   (a) Find the following values (the first may take a little effort):

$$\begin{aligned} f(0) &= f(f(5)) = \\ f(1) &= f(f(6)) = \\ f(2) &= f(f(7)) = \\ f(3) &= f(f(8)) = \\ f(4) &= f(f(9)) = \\ f(5) &= 1; \end{aligned}$$

   $f(6) = 2;$

   $f(7) = 3;$

   $f(8) = 4;$

   $f(9) = 5;$

   $\vdots$

   (b) Give a simpler recursive definition of the function $f : \mathbb{N} \to \mathbb{N}$ studied in Part (a).

   Basis:

$$f(0) = f(1) = f(2) = f(3) = f(4) = f(5) = 1;$$

Induction:

$$f(n) = f(n-1) + 1, \quad \text{if} \quad n \geq 6.$$