QUIZ 7 SOLUTIONS - CSCI 341

Friday, October 19

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Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Give the formal inductive definition of the set $\mathcal{R}$ of **regular expressions**:

   Basis: $\emptyset, \Lambda, a \in \mathcal{R}$, where $a \in A$;

   Induction: If $R, S \in \mathcal{R}$, then

   (a) $(R) \in \mathcal{R}$;
   (b) $R + S \in \mathcal{R}$;
   (c) $R \cdot S \in \mathcal{R}$; and
   (d) $R^* \in \mathcal{R}$.

2. Apply the operator L that associates to a given regular expression the corresponding regular language recursively (**showing all steps**) to discover the regular language $L(b(a^*bc^* + ac))$:

$$\begin{aligned}
L(b(a^*bc^* + ac)) &= L(b)L(a^*bc^* + ac)\\
&= L(b)(L(a^*bc^*) \cup L(ac))\\
&= L(b)(L(a)^*L(b)L(c)^* \cup L(a)L(c))\\
&= \{b\}(\{a\}^*\{b\}\{c\}^* \cup \{a\}\{c\})\\
&= \{b\}(\{a^n : n \geq 0\}\{b\}\{c^m : m \geq 0\} \cup \{ac\})\\
&= \{b\}(\{a^n bc^m : n, m \geq 0\} \cup \{ac\})\\
&= \{bac\} \cup \{ba^n bc^m : n, m \geq 0\}.
\end{aligned}$$

3. Write a regular expression for the language L over the alphabet $A = \{a, b, c\}$ consisting of all strings that contain the substring aba and end in c.

   The required expression is
   $$(a + b + c)^*aba(a + b + c)^*c.$$

4. The equation $(RR)^* = R^*R^*$ between regular expressions is <u>false</u>

   Proof:

   For alphabet $A = \{a\}$ and $R = a$, the given equation becomes
   $$(aa)^* = a^*a^*.$$
   Since $a \in L(a^*a^*)$, but $a \notin L((aa)^*)$, we have that
   $$L((aa)^*) \neq L(a^*a^*).$$
   Therefore, we conclude that
   $$(aa)^* \neq a^*a^*.$$