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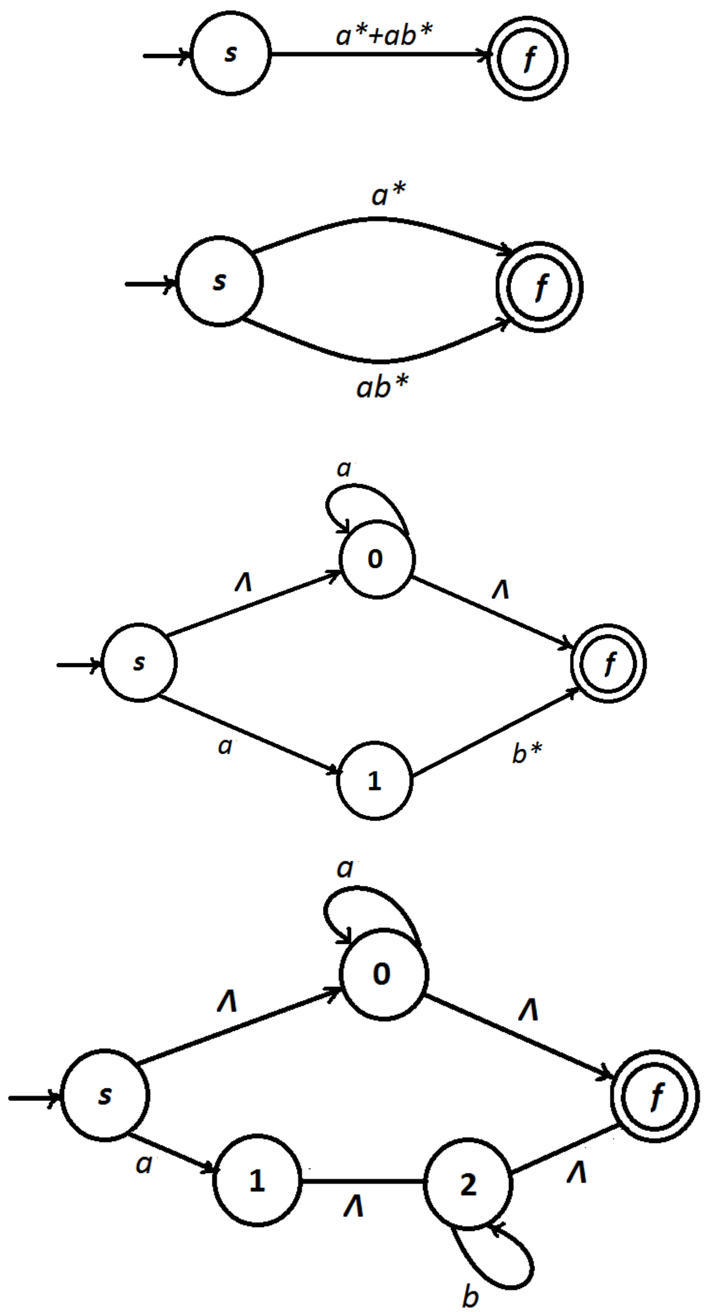
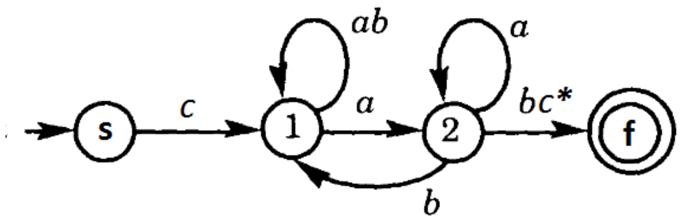
Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Use the top-down algorithm to construct an NFA for the regular language described by the regular expression $a^* + ab^*$ (show one step at a time; so you must execute 4 steps).



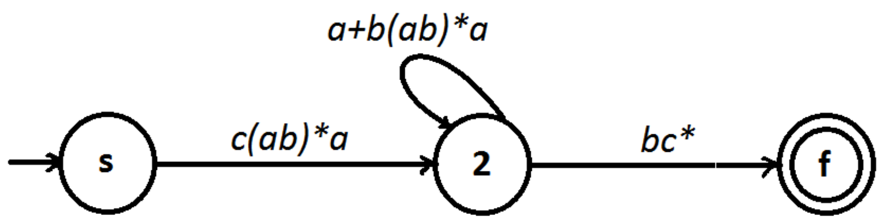
2. In the process of obtaining a regular expression for the regular language accepted by an NFA, a colleague of yours has obtained the following diagram:



An emergency has interrupted his work and you are called to continue the process. Execute the remaining two steps by eliminating first State 1 and then State 2 to get the final regular expression (you do not have to simplify after each step).

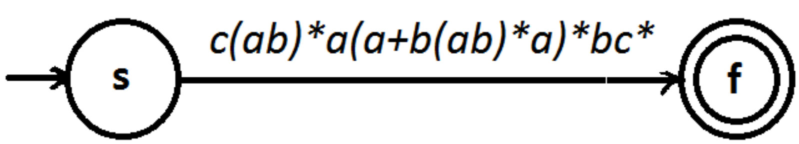
We eliminate state 1:

$$\begin{aligned} \text{new}(s,2) &= c(ab)^*a \\ \text{new}(2,2) &= a + b(ab)^*a \end{aligned}$$

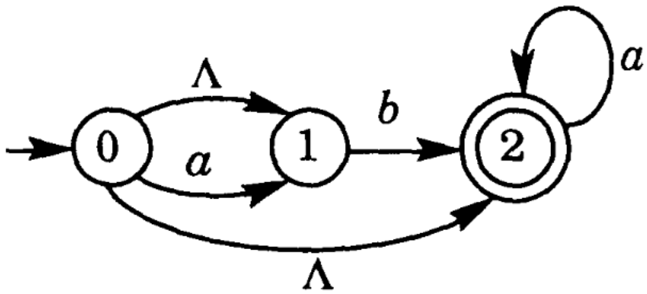


We eliminate state 2:

$$\text{new}(s,f) = c(ab)^*a[a + b(ab)^*a]^*bc^*$$



3. Use the algorithm described in class to get a DFA accepting the same regular language that is accepted by the NFA shown in the following picture. Please carry out one step at a time.



Start state: $\lambda(0) = \{0, 1, 2\}$

Transitions out of state $\lambda(0)$:

$$\begin{aligned} \delta(\{0,1,2\}, a) &= \lambda(\{1,2\}) = \{1,2\}; \\ \delta(\{0,1,2\}, b) &= \lambda(\{2\}) = \{2\}. \end{aligned}$$

Transitions out of state $\{1, 2\}$:

$$\begin{aligned} \delta(\{1,2\}, a) &= \lambda(\{2\}) = \{2\}; \\ \delta(\{1,2\}, b) &= \lambda(\{2\}) = \{2\}. \end{aligned}$$

Transitions out of state $\{2\}$:

$$\begin{aligned} \delta(\{2\}, a) &= \lambda(\{2\}) = \{2\}; \\ \delta(\{2\}, b) &= \lambda(\emptyset) = \emptyset. \end{aligned}$$

Transitions out of state $\emptyset$: $\delta(\emptyset, a) = \delta(\emptyset, b) = \emptyset$.

So we get the transition table shown below on the left, with $\{0, 1, 2\}$ the start state and all states except $\emptyset$ being final states.

After renaming the states to simplify, we get the transition table shown on the right.

State	a	b
$\{0,1,2\}$	$\{1,2\}$	$\{2\}$
$\{1,2\}$	$\{2\}$	$\{2\}$
$\{2\}$	$\{2\}$	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$

State	a	b
0	1	2
1	2	2
2	2	3
3	3	3

Finally, we give the diagram of the DFA just constructed: