EXAM 4 - MATH 111 Friday, April 8

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. A tank of water is contaminated with 60 lbs of salt. In order to bring the salt concentration down to a level consistent with EPA standards, clean water is being piped into the tank and the overflow mixture is collected for removal into a toxic-waste site. The result is that at the end of each hour there is 22% less salt in the tank than at the beginning of the hour. Let $S = S(t)$ be the number of pounds of salt in the tank t hours after the flushing process begins.

   (a) Explain why S is an exponential function and find its hourly decay factor.

   (b) Find a formula for $S(t)$.

   (c) Make a graph of S that shows the flushing process during the first 15 hours.

   (d) In order to meet EPA standards there can be no more than 3 lbs of salt in the tank. How long must the process continue before EPA standards are met?

   (e) Suppose that cleanup costs $ 8,000 per hour. How much does it cost to reduce the amount of salt from 60 lbs to 3 lbs? How much does it cost to reduce the amount of salt from 3 lbs to 0.1 lb?

2. The following table shows the number in millions of cell phone subscribers in the United States at the end of the given year:

Year	2001	2002	2003	2004	2005
Subscribers (millions)	128.4	140.8	158.7	182.1	207.9

(a) Are the data presented in the table exponential? Explain.

(b) Use exponential regression to construct an exponential model for the data. Indicate clearly the meaning of your variables and associated units.

(c) Make a sketch of the data points and of the model on the same system of coordinate axis.

(d) What was the yearly percentage growth rate from the end of 2001 through the end of 2005 for cell phone subscribership?

3. (a) Break the following logarithm into a sum/difference of logarithms:

$$\ln \frac{A^2\sqrt{B}}{C^3}$$

(b) Solve by hand the exponential equation

$$3^{t+5} = 7^{t-1}.$$

4. Stars have an **apparent magnitude** m, which is the brightness of light reaching Earth. They also have an **absolute magnitude** M, which is the intrinsic brightness and does not depend on the distance from the Earth. The difference $S = m - M$ is the **spectroscopic parallax**. Spectroscopic parallax is related to the distance D from the Earth, in parsecs, by

$$S = 5 \log D - 5.$$

(a) The distance to the star Kaus Astralis is 38.04 parsecs. What is its spectroscopic parallax?

(b) The spectroscopic parallax for the star Rasalhague is 1.27. How far away is Rasalhague from the Earth?

(c) How is the spectroscopic parallax affected when distance is multiplied by 10?

(d) The star Shaula is 3.78 times as far away as the star Atria. How does the spectroscopic parallax of Shaula compare to that of Atria?

5. In this problem follow the instructions carefully. The following table shows the length L in inches at age t in years of the North Sea sole (a type of fish):

t	1	2	3	4	5	6	7	8
L	3.7	7.5	10.0	11.5	12.7	13.5	14.0	14.4

The maximum length attained by the sole is 14.8 inches.

(a) Make a table showing for each age the difference D between the maximum length and the actual length L of the sole.

(b) Create the table of logarithms $\ln D$ of the differences for the various ages.

(c) Use linear regression to find a model of $\ln D$ vs. t.

(d) Use the model of the previous part to find a model for D vs. t.

(e) Find a formula expressing L as a function of t and plot the graph of the model L vs t.