

YOUR NAME: _____

George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Verify the following trigonometric identity:

$$\frac{1}{1 + \cos x} - \frac{1}{1 - \cos x} = -2 \cot x \csc x.$$

(**Hint:** Start from the left-hand side.)

2. Given $\cos \alpha = -\frac{3}{5}$, α in Quadrant III, and $\sin \beta = \frac{5}{13}$, β in Quadrant I, find $\sin(\alpha - \beta)$ and $\cos(\alpha + \beta)$.

3. If $\sin \alpha = -\frac{3}{5}$ and $\tan \alpha > 0$, compute the values $\sin 2\alpha$, $\cos 2\alpha$ and $\cos \frac{\alpha}{2}$.

4. Verify the identity

$$\frac{\cos(\alpha - \beta)}{\sin(\alpha + \beta)} = \frac{\cot \alpha + \tan \beta}{1 + \cot \alpha \tan \beta}.$$

5. (a) A given trigonometric equation is *not* an identity if there exists a value for the variable x for which the left side of the equation does not equal to the right side.

Verify that the following equation is **not** an identity by finding such an x -value:

$$\sqrt{1 + \tan^2 x} = \sec x.$$

- (b) Verify the following trigonometric identity

$$\sin^2 \frac{x}{2} = \frac{\sec x - 1}{2 \sec x}.$$

(**Hint:** Start from the right-hand side.)