

YOUR NAME: _____

George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Compute the following limits:

(a) $\lim_{x \rightarrow +\infty} \frac{\ln \ln x}{x}$

(b) $\lim_{x \rightarrow 0^+} (\sin 2x)^x$

2. Consider the function $f(x) = x^3 + 6x^2 + 9x$.

(a) Compute its first derivative and find all its critical points.

(b) Compute its second derivative and find all its critical points.

(c) Create the sign table including information about the signs of both derivatives and draw conclusions about monotonicity, concavity, extrema and inflection points of f . Show all your work and all this information very clearly.

(d) Based on the information of the previous part, draw a rough sketch of the graph of $y = f(x)$ indicating all points of interest.

3. Consider the function $f(x) = xe^{-x^2}$.

(a) Compute its first derivative and find all its critical points.

(b) Compute its second derivative and find all its critical points.

(c) Create the sign table including information about the signs of both derivatives and draw conclusions about monotonicity, concavity, extrema and inflection points of f . Show all your work and all this information very clearly.

(d) Based on the information of the previous part, draw a rough sketch of the graph of $y = f(x)$ indicating all points of interest.

- Find the points on the graph of the parabola $y^2 = x$ that are the closest to the point $(1, 0)$.

5. A car is traveling at 100 km/hr when the driver sees an accident 80 m ahead and slams on the brakes. What constant deceleration is required to stop the car in time to avoid a pileup?