EXAM 4 - MATH 251 Friday, April 12

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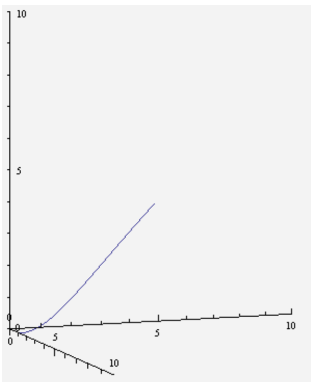
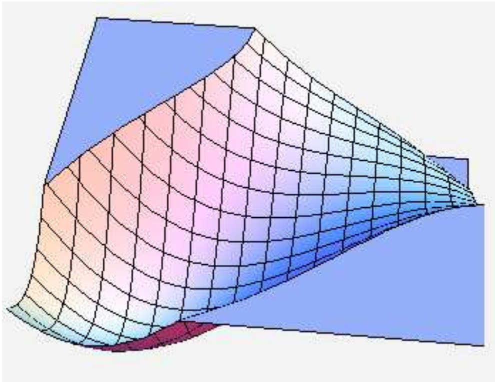
Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Suppose that a thermo-detecting robot, equipped with an ultra sensitive thermometer, is moving in a three dimensional space. At a point (x, y, z) in the space, the "temperature" is given by $T(x, y, z) = xye^z$. If at time t, the position of the robot is given by $\mathbf{r}(t) = \langle e^t, t, t^2 \rangle$, how fast is the temperature detected by the robot changing at $t = 1$ time unit into its motion?



2. Find the directional derivative of $f(x, y, z) = x \ln(y + z)$ at $P(2, e, e)$ in the direction of the vector $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$.

3. Find all local extrema and saddle points of $f(x, y) = x^3 + y^3 - 12xy$.



4. Find the volume of the solid enclosed by the paraboloid $z = x^2 + 3y^2$ and the planes $x = 0, y = 1, y = x$ and $z = 0$.

5. Use polar coordinates to find the volume of the solid inside both the cylinder $x^2 + y^2 = 4$ and the ellipsoid $4x^2 + 4y^2 + z^2 = 64$.