MATHEMATICAL LOGIC - MATH 300   George Voutsadakis
TAKE HOME EXAM 1   Due: Friday, March 8, 12:00 pm

YOUR NAME:_____ 🙂

It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. **You are not supposed to consult external sources** (besides our textbook and slides and your notes) and **you are not supposed to either provide help to or get help from anyone** inside or outside our class.

1. Use Boole's method of equational reasoning to establish the validity of the following argument:

$$A'(B' \cup C') = 0$$
$$CD' = 0$$
$$DE = 0$$
$$\therefore \ A'E = 0$$

2. Use Carroll's tree method to show the validity of the following argument:

$$ABF = 0$$
$$ACD = 0$$
$$B'D'E' = 0$$
$$C'DE' = 0$$
$$D'E'F' = 0$$
$$\therefore \ AE' = 0$$

3. Determine whether the following argument is valid

$$P \rightarrow Q$$
$$(Q \vee R) \wedge \neg(Q \wedge R)$$
$$\therefore \quad \neg Q \rightarrow (\neg P \wedge R)$$

4. In this problem, we are going to use the following notation:

N "needs to sleep 12 hours"
L "is the life of a party"
S "is a skier"
J "likes junk food"
T "likes TV soap operas"
M "rides a motorcycle".

Convert the statements in the following argument into propositional formulas and, then, determine whether the argument is valid, if the universe of discourse is the set of all people:

Those who do not need 12 hours sleep are the life of a party.
Those who need 12 hours sleep are not skiers nor do they like junk food.
Anyone who is not a skier likes TV soap operas.
A person who is the life of a party does not like TV soap operas nor ride a motorcycle.
Skiers do not like junk food.
$\therefore$ Everyone likes TV soap operas and rides a motorcycle.

5. **An Application of Compactness** 🙂

> **König's Infinity Lemma:** If a tree contains infinitely many vertices, each having finitely many children, then it has at least one infinite path.

Use carefully the Compactness Theorem to prove König's Lemma.

(**Hint:** Introduce for every vertex v in the tree a propositional variable P_v. The intuition is that P_v will be assigned the truth value 1 if and only if v is in the infinite path whose existence is asserted in the conclusion of the statement.)