HOMEWORK 1 - MATH 300 Due: Monday, January 21

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Read each problem **very carefully** and try to understand it well before starting to solve it. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. Write your own solutions and be neat!! **Take pride in your work!! Do not hand in scratchy doodles.**

1. Is $2 \in \{1,2,3\}$? Why?
2. Is $\{1,2\} \in \{\{1,2,3\},\{1,3\},1,2\}$? Why?
3. Give precise descriptions in plain English of the following sets:
   (a) $\{x \in \mathbb{N} : x \text{ is divisible by 2 and } x \text{ is divisible by 3}\}$
   (b) $\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$
   (c) $\{(x,y) \in \mathbb{R}^2 : y = 2x \text{ and } y = 3x\}$
4. Show formally the following statements:
   (a) $\{k \in \mathbb{Z} : k = 6m \text{ for some } m \in \mathbb{Z}\} \subseteq \{k \in \mathbb{Z} : k = 2n \text{ for some } n \in \mathbb{Z}\}$;
   (b) If $A \subsetneqq B$ and $B \subseteq C$, then $A \subsetneqq C$.
5. Is (each of) the following statement true for all sets A, B and C? If it is, give a proof. If it is not, provide a counterexample.
   (a) If $A \neq B$ and $B \neq C$, then $A \neq C$;
   (b) If $A \in B$ and $B \not\subseteq C$, then $A \notin C$;
   (c) If $A \subsetneqq B$ and $B \subseteq C$, then $C \not\subseteq A$;
6. Show that, for a set A in a universe U, we have $(A')' = A$.
7. Show that, for any sets A, B in a universe U, we have $(A \cup B)' = A' \cap B'$.
8. Either prove or give a counterexample for the following statement: For all sets A, B, C in a universe U, $(A \backslash B) \backslash C = A \backslash (B \cup C)$.
9. Consider the following three syllogisms:

(a)	All S is M	(b)	Some M is not P	(c)	All M is P
	No M is P		No M is S		Some S is M
	$\therefore$ Some S is P		$\therefore$ No S is P		$\therefore$ Some S is not P

   For each of (a),(b) and (c) provide its mood, its figure and explain whether it is a valid syllogism under the modern convention regarding the empty class.
10. Consider the following arguments

ARGUMENT 1	ARGUMENT 2
$(A \cup C')' = 0$	$(A' \cup C' \cup D)' = 0$
$(A'C)'(BC)' = 0$	$AD = 0$
$\therefore\ (BC')' = 0$	$BC' = 0$
	$\therefore\ AB = 0$

   (a) Use a Venn diagram to determine if each argument is correct.
   (b) If the argument is correct, then use both Boole's equational reasoning and Carroll's tree method to prove its correctness.