HOMEWORK 2 - MATH 300 Due: Monday, February 11

YOUR NAME:_____ George Voutsadakis

Read each problem **very carefully** and try to understand it well before starting to solve it. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. Write your own solutions and be neat!! **Take pride in your work!! Do not hand in scratchy doodles.**

1. Consider the valuation $\mathbf{e} = (1, 0, 0, 1)$ for the propositional variables P, Q, R, S. For each of the following formulas F, show $F(\mathbf{e})$ and find $\hat{F}(\mathbf{e})$.
   (a) $R \to (S \wedge P)$;
   (b) $P \to (R \to S)$;
   (c) $(P \vee R) \leftrightarrow (R \wedge \neg S)$;
   (d) $(Q \wedge \neg S) \to (P \leftrightarrow S)$;
   (e) $R \wedge S \to (P \to \neg Q \vee S)$;
   (f) $(P \vee \neg Q) \vee R \to (S \wedge \neg S)$.

2. Construct truth tables for each of the following formulas:
   (a) $P \to (P \to Q)$;
   (b) $P \to \neg(Q \wedge R)$;
   (c) $(P \to Q) \leftrightarrow \neg P \vee Q$;
   (d) $P \wedge Q \to (Q \wedge \neg Q \to R \wedge Q)$.

3. For each of the following formulas determine whether the information given is sufficient to decide its truth value. If it is, state that truth value; if it is not, show that both truth values are possible.
   (a) $(P \to Q) \to R$; R is assigned truth value 1;
   (b) $P \vee (Q \to R)$; truth value of $Q \to R$ is 1;
   (c) $(P \to Q) \to (\neg Q \to \neg P)$; Q is assigned truth value 1;
   (d) $(P \wedge Q) \to (P \vee S)$; P is assigned truth value 1 and S truth value 0;

4. The following statement

   "If labor or management is stubborn, then the strike will be settled if and only if the government obtains an injunction, but troops are not sent into the factory"

   may be expressed as a formula $L \vee M \to (S \leftrightarrow G \wedge \neg R)$, with the obvious implied meaning for the propositional variables involved.
   (a) Determine the truth value of the given statement under the assumption that both labor and management are stubborn, the strike will not be settled, the government fails to obtain an injunction and troops are sent into the factory.
   (b) Determine the truth value of the given statement if it is agreed that

   "If the government obtains an injunction, then troops will be sent into the factory. If troops are sent into the factory, then the strike will not be settled. The strike will be settled. The management is stubborn."

5. Which pairs of the following propositional formulas are truth equivalent? (Show all work.)

   (a) $\neg(P \leftrightarrow (R \leftrightarrow P))$

   (b) $P \vee ((P \leftrightarrow R) \vee P)$

   (c) $R \vee ((\neg Q \leftrightarrow P) \leftrightarrow Q)$

   (d) $(R \rightarrow (\neg P \rightarrow P)) \vee P$

   (e) $(R \leftrightarrow P) \vee ((P \vee (Q \vee R)) \rightarrow P)$

6. Which of the following propositional formulas are tautologies and which contradictions? (Show all work.)

   (a) $((P \leftrightarrow ((\neg Q) \vee R)) \rightarrow ((\neg P) \rightarrow Q))$

   (b) $((P \rightarrow (Q \vee R)) \vee (P \rightarrow Q))$

   (c) $((P \leftrightarrow Q) \leftrightarrow (P \leftrightarrow (Q \leftrightarrow P)))$

   (d) $((P \vee (\neg(Q \wedge R))) \rightarrow ((P \leftrightarrow R) \vee Q))$

   (e) $(\neg((\neg R) \rightarrow ((S \wedge Q) \rightarrow S)))$

7. Let F, G be propositional formulas. Show that, if the formulas F and $(F \rightarrow G)$ are tautologies, then so is G.

8. Show that the set of connectives $\{\neg, \leftrightarrow\}$ is not adequate. (**Hint:** Study the work we did with $\rightarrow$ in class!)

9. In class, we showed that $\{\barwedge\}$ and $\{|\ \}$ are adequate sets of connectives. Express the propositional formula $P \wedge (Q \rightarrow R)$ in terms of $|$. (Please, do not just write a single formula; provide a couple of steps with explanation on the way!)

10. Find the disjunctive normal forms, first using rewriting rules (the $\rightsquigarrow$-rules of the slides), and, then, using the method of truth tables.

    (a) $(P \rightarrow Q) \rightarrow (Q \rightarrow R)$

    (b) $Q \rightarrow (Q \rightarrow (R \rightarrow S))$