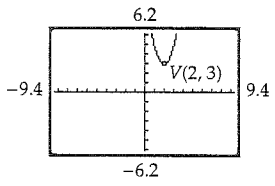
2. Examine the following four equations and the graphs labeled **i, ii, iii,** and **iv.** Determine which graph is the graph of each equation.

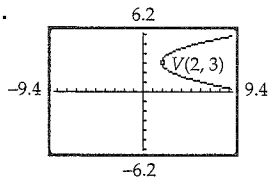
a. $(y-3)^2 = x-2$ **b.** $(y-3)^2 = -2(x-2)$

c. $\frac{1}{2}(y-3) = (x-2)^2$ **d.** $-2(y-3) = (x-2)^2$

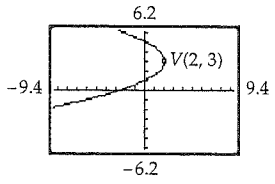
i.



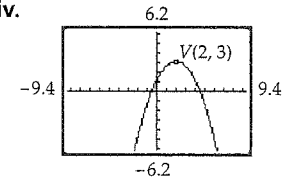
ii.



iii.



iv.



In Exercises 3 to 28, find the vertex, focus, and directrix of the parabola given by each equation. Sketch the graph.

3. $x^2 = -4y$ **4.** $2y^2 = x$

5. $y^2 = \frac{1}{3}x$ **6.** $x^2 = -\frac{1}{4}y$

7. $(x-2)^2 = 8(y+3)$ **8.** $(y+1)^2 = 6(x-1)$

9. $(y+4)^2 = -4(x-2)$ **10.** $(x-3)^2 = -(y+2)$

11. $(y-1)^2 = 2x+8$ **12.** $(x+2)^2 = 3y-6$

13. $(2x-4)^2 = 8y-16$ **14.** $(3x+6)^2 = 18y-36$

15. $x^2+8x-y+6=0$ **16.** $x^2-6x+y+10=0$

17. $x+y^2-3y+4=0$ **18.** $x-y^2-4y+9=0$

19. $2x-y^2-6y+1=0$ **20.** $3x+y^2+8y+4=0$

21. $x^2+3x+3y-1=0$ **22.** $x^2+5x-4y-1=0$

23. $2x^2-8x-4y+3=0$ **24.** $6x-3y^2-12y+4=0$

25. $2x+4y^2+8y-5=0$ **26.** $4x^2-12x+12y+7=0$

27. $3x^2-6x-9y+4=0$ **28.** $2x-3y^2+9y+5=0$

29. Find the equation in standard form of the parabola with vertex at the origin and focus $(0, -4)$.

30. Find the equation in standard form of the parabola with vertex at the origin and focus $(5, 0)$.

31. Find the equation in standard form of the parabola with vertex at $(-1, 2)$ and focus $(-1, 3)$.

32. Find the equation in standard form of the parabola with vertex at $(2, -3)$ and focus $(0, -3)$.

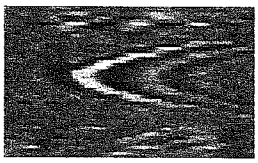
33. Find the equation in standard form of the parabola with focus $(3, -3)$ and directrix $y = -5$.

34. Find the equation in standard form of the parabola with focus $(-2, 4)$ and directrix $x = 4$.

35. Find the equation in standard form of the parabola that has vertex $(-4, 1)$, has its axis of symmetry parallel to the y-axis, and passes through the point $(-2, 2)$.

36. Find the equation in standard form of the parabola that has vertex $(3, -5)$, has its axis of symmetry parallel to the x-axis, and passes through the point $(4, 3)$.

37. STRUCTURAL DEFECTS Ultrasound is used as a nondestructive method of determining whether a support beam for a structure has an internal fracture. In one scanning procedure, if the resulting image is a parabola, engineers know that there is a structural defect. Suppose that a scan produced an image whose equation is



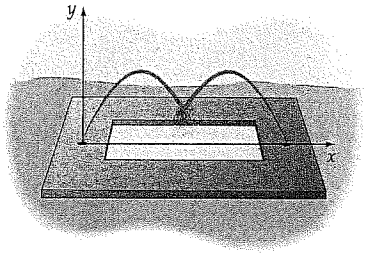
$$x = -0.325y^2 + 13y + 120$$

Determine the vertex and focus of the graph of this parabola.

38. FOUNTAIN DESIGN A fountain in a shopping mall has two parabolic arcs of water intersecting as shown below. The equation of one parabola is $y = -0.25x^2 + 2x$ and the equation of the second parabola is

$$y = -0.25x^2 + 4.5x - 16.25$$

How high above the base of the fountain do the parabolas intersect? All dimensions are in feet.



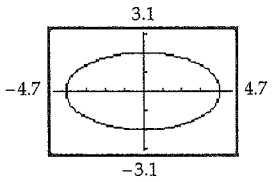
Exercise Set 6.2

1. Examine the following four equations and the graphs labeled **i, ii, iii,** and **iv.** Determine which graph is the graph of each equation.

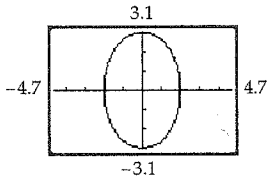
a. $\dfrac{(x+1)^2}{9}+\dfrac{(y-2)^2}{1}=1$ **b.** $\dfrac{x^2}{16}+\dfrac{y^2}{4}=1$

c. $\dfrac{x^2}{4}+\dfrac{y^2}{9}=1$ **d.** $\dfrac{(x+1)^2}{9}+\dfrac{(y-2)^2}{16}=1$

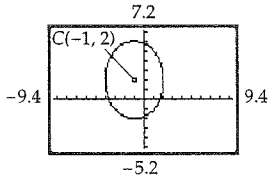
i.



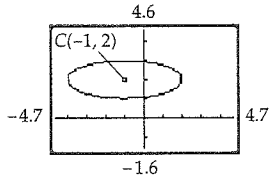
ii.



iii.



iv.

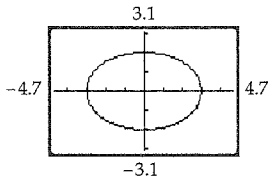


2. Examine the following four equations and the graphs labeled **i, ii, iii,** and **iv.** Determine which graph is the graph of each equation.

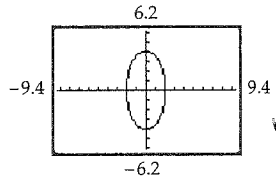
a. $\dfrac{(x-3)^2}{1}+\dfrac{(y-2)^2}{4}=1$ **b.** $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$

c. $\dfrac{(x-3)^2}{25}+\dfrac{(y-2)^2}{16}=1$ **d.** $\dfrac{x^2}{4}+\dfrac{y^2}{16}=1$

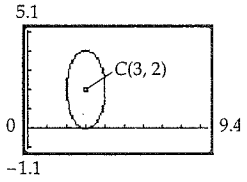
i.



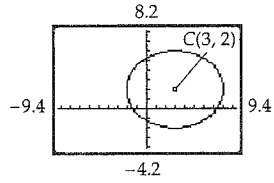
ii.



iii.



iv.



In Exercises 3 to 34, find the center, vertices, and foci of the ellipse given by each equation. Sketch the graph.

3. $\dfrac{x^2}{16}+\dfrac{y^2}{25}=1$

4. $\dfrac{x^2}{49}+\dfrac{y^2}{36}=1$

5. $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$

6. $\dfrac{x^2}{64}+\dfrac{y^2}{25}=1$

7. $\dfrac{x^2}{9}+\dfrac{y^2}{7}=1$

8. $\dfrac{x^2}{5}+\dfrac{y^2}{4}=1$

9. $\dfrac{4x^2}{9}+\dfrac{y^2}{16}=1$

10. $\dfrac{x^2}{9}+\dfrac{9y^2}{16}=1$

11. $\dfrac{(x-3)^2}{25}+\dfrac{(y+2)^2}{16}=1$

12. $\dfrac{(x+3)^2}{9}+\dfrac{(y+1)^2}{16}=1$

13. $\dfrac{(x+2)^2}{9}+\dfrac{y^2}{25}=1$

14. $\dfrac{x^2}{25}+\dfrac{(y-2)^2}{81}=1$

15. $\dfrac{(x-1)^2}{21}+\dfrac{(y-3)^2}{4}=1$

16. $\dfrac{(x+5)^2}{9}+\dfrac{(y-3)^2}{7}=1$

17. $\dfrac{9(x-1)^2}{16}+\dfrac{(y+1)^2}{9}=1$

18. $\dfrac{(x+6)^2}{25}+\dfrac{25y^2}{144}=1$

19. $3x^2+4y^2=12$

20. $5x^2+4y^2=20$

21. $25x^2+16y^2=400$

22. $25x^2+12y^2=300$

23. $64x^2+25y^2=400$

24. $9x^2+64y^2=144$

25. $4x^2+y^2-24x-8y+48=0$

26. $x^2+9y^2+6x-36y+36=0$

27. $5x^2+9y^2-20x+54y+56=0$

28. $9x^2+16y^2+36x-16y-104=0$

29. $16x^2+9y^2-64x-80=0$

30. $16x^2+9y^2+36y-108=0$

31. $25x^2+16y^2+50x-32y-359=0$

32. $16x^2+9y^2-64x-54y+1=0$

33. $8x^2 + 25y^2 - 48x + 50y + 47 = 0$

34. $4x^2 + 9y^2 + 24x + 18y + 44 = 0$

In Exercises 35 to 46, find the equation in standard form of each ellipse, given the information provided.

35. Center $(0, 0)$, major axis of length 10, foci at $(4, 0)$ and $(-4, 0)$

36. Center $(0, 0)$, minor axis of length 6, foci at $(0, 4)$ and $(0, -4)$

37. Vertices $(6, 0)$, $(-6, 0)$; ellipse passes through $(0, -4)$ and $(0, 4)$

38. Vertices $(7, 0)$, $(-7, 0)$; ellipse passes through $(0, 5)$ and $(0, -5)$

39. Major axis of length 12 on the x-axis, center at $(0, 0)$; ellipse passes through $(2, -3)$

40. Major axis of length 8, center at $(0, 0)$; ellipse passes through $(-2, 2)$

41. Center $(-2, 4)$, vertices $(-6, 4)$ and $(2, 4)$, foci at $(-5, 4)$ and $(1, 4)$

42. Center $(0, 3)$, minor axis of length 4, foci at $(0, 0)$ and $(0, 6)$

43. Center $(2, 4)$, major axis parallel to the y-axis and of length 10; ellipse passes through the point $(3, 3)$

44. Center $(-4, 1)$, minor axis parallel to the y-axis and of length 8; ellipse passes through the point $(0, 4)$

45. Vertices $(5, 6)$ and $(5, -4)$, foci at $(5, 4)$ and $(5, -2)$

46. Vertices $(-7, -1)$ and $(5, -1)$, foci at $(-5, -1)$ and $(3, -1)$

In Exercises 47 to 54, use the eccentricity of each ellipse to find its equation in standard form.

47. Eccentricity $\frac{2}{5}$, major axis on the x-axis and of length 10, center at $(0, 0)$

48. Eccentricity $\frac{3}{4}$, foci at $(9, 0)$ and $(-9, 0)$

49. Foci at $(0, -4)$ and $(0, 4)$, eccentricity $\frac{2}{3}$

50. Foci at $(0, -3)$ and $(0, 3)$, eccentricity $\frac{1}{4}$

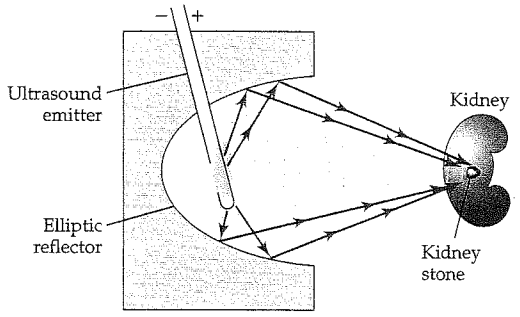
51. Eccentricity $\frac{2}{5}$, foci at $(-1, 3)$ and $(3, 3)$

52. Eccentricity $\frac{1}{4}$, foci at $(-2, 4)$ and $(-2, -2)$

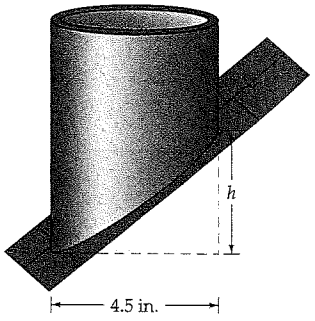
53. Eccentricity $\frac{2}{3}$, major axis of length 24 on the y-axis, center at $(0, 0)$

54. Eccentricity $\frac{3}{5}$, major axis of length 15 on the x-axis, center at $(0, 0)$

55. MEDICINE A *lithotripter* is an instrument used to remove a kidney stone in a patient without having to do surgery. A high-frequency sound wave is emitted from a source that is located at the focus of an ellipse. The patient is placed so that the kidney stone is located at the other focus of the ellipse. If the equation of the ellipse is $\frac{(x-11)^2}{484} + \frac{y^2}{64} = 1$ (x and y are measured in centimeters), where, to the nearest centimeter, should the patient's kidney stone be placed so that the reflected sound hits the kidney stone?



56. CONSTRUCTION A circular vent pipe is placed on a roof that has a slope of $\frac{4}{5}$, as shown in the figure at the right.



a. Use the slope to find the value of h.

b. The intersection of the vent pipe and the roof

Exercise Set 6.3

1. Examine the following four equations and the graphs labeled **i, ii, iii,** and **iv.** Determine which graph is the graph of each equation.

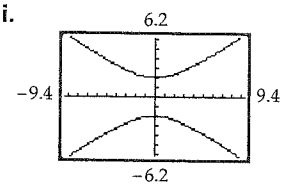
a. $\frac{(x-1)^2}{9} - \frac{(y-2)^2}{16} = 1$

b. $\frac{x^2}{9} - \frac{y^2}{4} = 1$

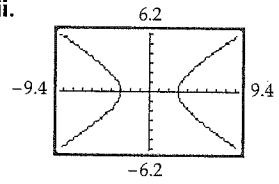
c. $\frac{y^2}{4} - \frac{x^2}{9} = 1$

d. $\frac{(y-2)^2}{4} - \frac{(x-1)^2}{9} = 1$

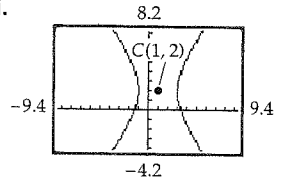
i.



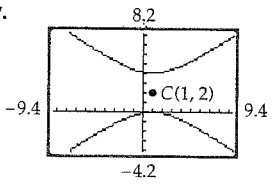
ii.



iii.



iv.



2. Examine the following four equations and the graphs labeled **i, ii, iii,** and **iv.** Determine which graph is the graph of each equation.

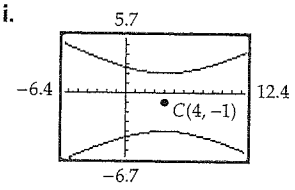
a. $\frac{x^2}{25} - \frac{y^2}{4} = 1$

b. $\frac{(y+1)^2}{9} - \frac{(x-4)^2}{36} = 1$

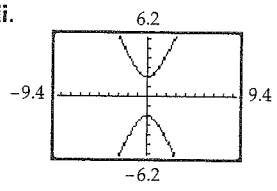
c. $\frac{(x-4)^2}{36} - \frac{(y+1)^2}{9} = 1$

d. $\frac{y^2}{4} - x^2 = 1$

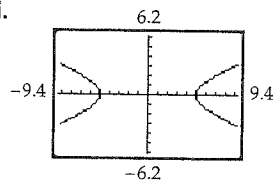
i.



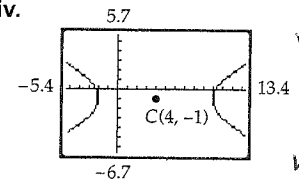
ii.



iii.



iv.



In Exercises 3 to 28, find the center, vertices, foci, and asymptotes for the hyperbola given by each equation. Graph each equation.

3. $\frac{x^2}{16} - \frac{y^2}{25} = 1$

4. $\frac{x^2}{16} - \frac{y^2}{9} = 1$

5. $\frac{y^2}{4} - \frac{x^2}{25} = 1$

6. $\frac{y^2}{25} - \frac{x^2}{36} = 1$

7. $\frac{x^2}{7} - \frac{y^2}{9} = 1$

8. $\frac{x^2}{5} - \frac{y^2}{4} = 1$

9. $\frac{4x^2}{9} - \frac{y^2}{16} = 1$

10. $\frac{x^2}{9} - \frac{9y^2}{16} = 1$

11. $\frac{(x-3)^2}{16} - \frac{(y+4)^2}{9} = 1$

12. $\frac{(x+3)^2}{25} - \frac{y^2}{4} = 1$

13. $\frac{(y+2)^2}{4} - \frac{(x-1)^2}{16} = 1$

14. $\frac{(y-2)^2}{36} - \frac{(x+1)^2}{49} = 1$

15. $\frac{(x+2)^2}{9} - \frac{y^2}{25} = 1$

16. $\frac{x^2}{25} - \frac{(y-2)^2}{81} = 1$

17. $\frac{9(x-1)^2}{16} - \frac{(y+1)^2}{9} = 1$

18. $\frac{(x+6)^2}{25} - \frac{25y^2}{144} = 1$

19. $x^2 - y^2 = 9$

20. $4x^2 - y^2 = 16$

21. $16y^2 - 9x^2 = 144$

22. $9y^2 - 25x^2 = 225$

23. $9y^2 - 36x^2 = 4$

24. $16x^2 - 25y^2 = 9$

25. $x^2 - y^2 - 6x + 8y - 3 = 0$

26. $4x^2 - 25y^2 + 16x + 50y - 109 = 0$

27. $9x^2 - 4y^2 + 36x - 8y + 68 = 0$

28. $16x^2 - 9y^2 - 32x - 54y + 79 = 0$

In Exercises 29 to 34, use the quadratic formula to solve for y in terms of x. Then use a graphing utility to graph each equation.

29. $4x^2 - y^2 + 32x + 6y + 39 = 0$

30. $x^2 - 16y^2 + 8x - 64y + 16 = 0$

31. $9x^2 - 16y^2 - 36x - 64y + 116 = 0$

32. $2x^2 - 9y^2 + 12x - 18y + 18 = 0$

33. $4x^2 - 9y^2 + 8x - 18y - 6 = 0$

34. $2x^2 - 9y^2 - 8x + 36y - 46 = 0$

In Exercises 35 to 48, find the equation in standard form of the hyperbola that satisfies the stated conditions.

35. Vertices $(3, 0)$ and $(-3, 0)$, foci $(4, 0)$ and $(-4, 0)$

36. Vertices $(0, 2)$ and $(0, -2)$, foci $(0, 3)$ and $(0, -3)$

37. Foci $(0, 5)$ and $(0, -5)$, asymptotes $y = 2x$ and $y = -2x$

38. Foci $(4, 0)$ and $(-4, 0)$, asymptotes $y = x$ and $y = -x$

39. Vertices $(0, 3)$ and $(0, -3)$, passing through $(2, 4)$

40. Vertices $(5, 0)$ and $(-5, 0)$, passing through $(-1, 3)$

41. Asymptotes $y = \frac{1}{2}x$ and $y = -\frac{1}{2}x$, vertices $(0, 4)$ and $(0, -4)$

42. Asymptotes $y = \frac{2}{3}x$ and $y = -\frac{2}{3}x$, vertices $(6, 0)$ and $(-6, 0)$

43. Vertices $(6, 3)$ and $(2, 3)$, foci $(7, 3)$ and $(1, 3)$

44. Vertices $(-1, 5)$ and $(-1, -1)$, foci $(-1, 7)$ and $(-1, -3)$

45. Foci $(1, -2)$ and $(7, -2)$, slope of an asymptote $\frac{5}{4}$

46. Foci $(-3, -6)$ and $(-3, -2)$, slope of an asymptote 1

47. Passing through $(9, 4)$, slope of an asymptote $\frac{1}{2}$, center $(7, 2)$, transverse axis parallel to the y-axis

48. Passing through $(6, 1)$, slope of an asymptote 2, center $(3, 3)$, transverse axis parallel to the x-axis

In Exercises 49 to 54, use the eccentricity to find the equation in standard form of each hyperbola.

49. Vertices $(1, 6)$ and $(1, 8)$, eccentricity 2

50. Vertices $(2, 3)$ and $(-2, 3)$, eccentricity $\frac{5}{2}$

51. Eccentricity 2, foci $(4, 0)$ and $(-4, 0)$

52. Eccentricity $\frac{4}{3}$, foci $(0, 6)$ and $(0, -6)$

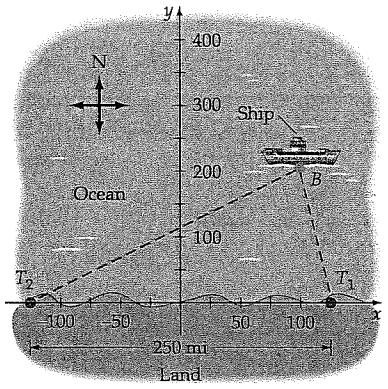
53. Center $(4, 1)$, conjugate axis of length 4, eccentricity $\frac{4}{3}$ (*Hint:* There are two answers.)

54. Center $(-3, -3)$, conjugate axis of length 6, eccentricity 2 (*Hint:* There are two answers.)

55. LORAN Two radio transmitters are positioned along the coast, 250 miles apart. A signal is sent simultaneously from each transmitter. The signal from transmitter T_2 is received by a ship's LORAN 500 microseconds after the ship receives the signal from T_1. The radio signals travel at 0.186 mile per microsecond.

a. Find an equation of a hyperbola, with foci at T_1 and T_2, on which the ship is located.

b. If the ship is 100 miles east of the y-axis, determine its distance from the coastline (to the nearest mile).



56. LORAN Two radio transmitters are positioned along the coast, 300 miles apart. A signal is sent simultaneously from each transmitter. The signal from transmitter T_1 is