

YOUR NAME: _____

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Find an equation for the tangent line to the graph of $f(x) = \frac{x^3 + 1}{x + 3}$ at $x = -2$.

2. The number of bacteria in a chemically treated culture is given by $N(t) = \sqrt[3]{(5t + 2)^2}$ (in hundreds), with t being the number of days since treatment began, $0 \leq t \leq 10$. Find the rate of change of the population at $t = 5$, give the unit, and interpret your answer.

3. Compute the following derivative explicitly mentioning the rule used at each step:

$$\left[\sqrt[5]{(2x^2 + 1)^4 + x^4} \right]' =$$

4. Consider the function

$$f(x) = x^4 + 4x^3 + 15.$$

(a) Compute the first derivative and find its critical points.

(b) Compute the second derivative and find its critical points.

(c) Create the combined sign table of the first and second derivative showing clearly on the last line intervals of monotonicity, intervals of concavity, relative extrema and inflection points (you do not have to graph).

5. Consider the function $f(x) = \frac{1}{x^2 - x - 2}$.

(a) Find the domain.

(b) Find the asymptotes (these must be equations of lines).

(c) Find the first derivative and its critical points.

(d) Construct the sign table for the first derivative and find intervals of monotonicity and relative extrema for f .

(e) Sketch the graph of f .