EXAM 1 - MATH 251
YOUR NAME:

Read each problem very carefully before starting to solve it. Each problem is worth 10 points. It is necessary to show all your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Consider the parametric curve $\left\{\begin{array}{ll}x & =\ln t \\ y & =3 t^{2}-t\end{array}\right.$. Find an equation for the tangent line to the curve at the point $P=(0,2)$.
2. Calculate the length of the portion of the polar curve $r=\theta^{2}$ shown in the figure:

3. Calculate the shaded area in the figure, where the polar curve is given by $r^{2}=\cos \theta e^{\sin \theta}$.

4. A hyperbola has vertices $(-1,5)$ and $(3,5)$ and one of its asymptotes is given by $y-5=$ $\frac{5}{2}(x-1)$.
(a) Find an equation for the hyperbola.
(b) Locate its foci.
5. (a) Find an equation in symmetric form for the line that passes through the points $P=$ $(-2,0,2)$ and $Q=(4,3,7)$.
(b) Determine whether the lines $\mathbf{r}_{1}(t)=\langle 2,1,1\rangle+t\langle-4,0,1\rangle$ and $\mathbf{r}_{2}(t)=\langle-4,1,5\rangle+$ $t\langle 2,1,-2\rangle$ intersect and, if yes, find the point of intersection.
