EXAM 3 - MATH 251 Friday, March 30

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Consider the function $f(x, y) = \sqrt{4 - x^2 - y^2} \ln(y - x)$.

   (a) Find its domain

   $$\mathcal{D} = \{(x, y) \in \mathbb{R}^2 : \qquad\qquad\qquad\qquad \}.$$

   (b) Carefully sketch the domain, labeling points and showing relevant details.

2. Show that $\displaystyle\lim_{(x,y)\to(-2,1)} \frac{y^2 - 1}{x^3 - y + 9}$ does not exist.

   (Please describe clearly the curves chosen and give their equations!)

3. Compute the following partial derivatives:

   (a) f_{xyz} if $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.

   (b) $g_{xy}(-3, 2)$ if $g(x, y) = xe^{-xy}$.

4. (a) Find an equation for the tangent plane to $f(x, y) = \ln(4x^2 - y^2)$ at the point $(1, 1)$.

(b) Assume

$$\begin{aligned} f(1,0,0) &= 3, \quad & f_x(1,0,0) &= -2, \\ f_y(1,0,0) &= 4, \quad & f_z(1,0,0) &= 2. \end{aligned}$$

Estimate the value of $f(1.02, 0.01, -0.03)$.

5. (a) Compute $\dfrac{d}{dt}f(\boldsymbol{c}(t))$, if $f(x, y) = \ln x + \ln y$ and $\boldsymbol{c}(t) = \langle \cos t, t^2 \rangle$, at the point $t = \frac{\pi}{4}$.

(b) Calculate the directional derivative of $f(x, y) = \sin(x - y)$ in the direction of $\boldsymbol{v} = \langle 1, 1 \rangle$ at the point $P = (\frac{\pi}{2}, \frac{\pi}{6})$.

(c) Find an equation for the tangent plane to the surface $x^2 + z^2 e^{y-x} = 13$ at the point $P = (2, 3, \frac{3}{\sqrt{e}})$.